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JEE MAIN

APRIL 2019

KCET

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MATHEMATICS today

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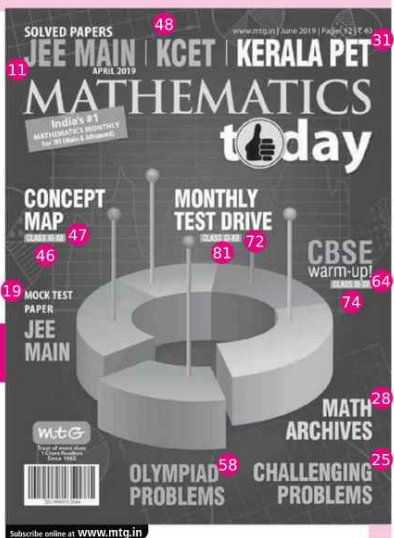
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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the brains of bright students seeking admission into IITs with additional study material. During the last 10 years there have been several changes in the pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefiting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India. Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers collaborating them. We do hope that students will continue to use Maths Musing to boost up their rank in JEE Main and Advanced.

PROBLEM Set 198

JEE MAIN

1. If ω is a cube root of unity, then

$$\cos \left[\frac{\{(1-\omega)(1-\omega^2)+\dots+(10-\omega)(10-\omega^2)\} \pi}{900} \right] =$$

(a) -1 (b) 0
(c) 1 (d) $\sqrt{3}/2$

2. The value of $\int_{-\pi/2}^{199\pi/2} \sqrt{1+\cos 2x} dx$ is

(a) $50\sqrt{2}$ (b) $100\sqrt{2}$
(c) $150\sqrt{2}$ (d) $200\sqrt{2}$

3. If two different tangents of $y^2 = 4x$ are the normals to $x^2 = 4by$, then

(a) $|b| > 1/2\sqrt{2}$ (b) $|b| < 1/2\sqrt{2}$
(c) $|b| > 1/\sqrt{2}$ (d) $|b| < 1/\sqrt{2}$

4. The function f satisfies the functional equation

$$3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30 \text{ for all real } x \neq 1.$$

The value of $f(7)$ is

(a) 8 (b) 4 (c) -8 (d) 11

5. The sum of $i - 2 - 3i + 4 \dots$ up to 100 terms, where $i = \sqrt{-1}$ is

(a) $50(1-i)$ (b) $25i$
(c) $25(1+i)$ (d) $100(1-i)$

JEE ADVANCED

6. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular and the unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c} = \alpha\vec{a} + \beta\vec{b} + \gamma(\vec{a} \times \vec{b})$, then

(a) $\alpha = \beta$ (b) $\gamma^2 = 1 - 2\alpha^2$
(c) $\gamma^2 = -\cos 2\theta$ (d) $\beta^2 = \frac{1+\cos 2\theta}{2}$

COMPREHENSION

Let $f(x) = \begin{cases} x+a, & x < 0 \\ |x-1|, & x \geq 0 \end{cases}$ and

$$g(x) = \begin{cases} x+1, & x < 0 \\ (x-1)^2+b, & x \geq 0 \end{cases}$$

7. The value of a , if $(g \circ f)x$ is continuous for all real x , is

(a) -1 (b) 0 (c) 1 (d) 2

8. The value of b , if $(g \circ f)x$ is continuous for all real x , is

(a) -1 (b) 0 (c) 1 (d) 2

NUMERICAL ANSWER TYPE

9. If the minimum value of the expression $y = (27)^{\cos x} + (81)^{\sin x}$ is m , then the value of $\{1/m\}$ is (where $\{ \cdot \}$ represents greatest integer function)

MATRIX MATCH

10. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, then match the columns :

	Column-I	Column-II
P.	$a_0 + a_1 + \dots + a_{2n} =$	1. $n+1$
Q.	$a_0 - a_1 + a_2 - \dots + a_{2n} =$	2. 3^n
R.	$a_0 - 2a_1 + 3a_2 - \dots + (2n+1)a_{2n} =$	3. 1

P	Q	R
(a) 3	1	2
(b) 1	2	3
(c) 3	2	1
(d) 2	3	1

See Solution Set of Maths Musing 197 on page no. 85

KNOWLEDGE SERIES

(for JEE / Olympiad Aspirants)

Myth

Competitive examination preparation is not helpful for school examination.

Reality

When you study for competitive examination, your knowledge base becomes strong and you go in depth of subject.

Suggestion - Before solving competitive exam books, first solve your school books and learn how to write solution for school exam.

Please visit youtube to watch more myth and reality discussions on KCS EDUCATE channel.

Check your concepts & win prizes.

Knowledge Quiz - 19

If $A \subset \{1, 2, 3, \dots, 100\}$, $|A| = 50$ such that no two numbers from A have their sum as 100. Show that A contains a square.

Please send your detailed solution before 15th June 2019 to quiz@kcseducate.in along with your name, father's name, class, school, address and contact number.

Winner Knowledge Quiz - 18

- Krishna N Agaram, (Class - X), Maharishi Vidya Mandir, Hyderabad, Telangana.
- Abhisarita Maity, (Class - XI), Bishnupur High School, Bankura, West Bengal.
- Sameekahyo Basu, (Class - XII), Garden High School, Kolkata, West Bengal.



RISHABH BHATNAGAR



CBSE Topper (C.G.) Class 12th



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Aaron Jerry Ninan
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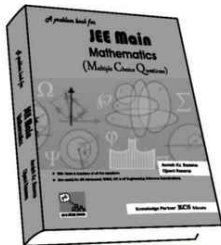
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Cracking the JEE Main EXAM



Shubham Srivastava

JEE Main 2019 Topper Shubham Srivastava (AIR 1)

"Delhi's Shubham Srivastava tops JEE Main 2019, found April exam easier than January"

Shubham Srivastava, a resident of Sector 22 in Dwarka, New Delhi has become Topper in the JEE Main exam 2019. He secured All India Rank (AIR) 1 by scoring 360 marks out of 360 in the exam.

Calling his daily routine his secret of success, Shubham Srivastava, said that he never got distracted by phone or games because that does not interest him and he gets joy out of solving Mathematics and Physics problems.

After scoring 99.96 percentile in his January attempt of the Joint Entrance Exam (JEE) Main 2019, Shubham Srivastava decided to give another try in April. Just like 2,97,932 candidates, Srivastava too had improved his score in the second attempt at JEE Main 2019. The 18-year-old, who was earlier, second to Naveen Jindal (who secured 99.99 percentile) as Delhi state topper, has secured All India Rank (AIR) 1.

Srivastava was contented with his score and appeared for the engineering entrance exam for the love of the subjects. "More than preparing for JEE, it was about studying my favourite subjects - Physics, Chemistry, and Mathematics. I feel happy while solving these questions, the more the difficulty-level the happier I am," he said. While many students claimed different difficulty levels in January and April exams, NTA claims to have maintained a standard level.

"I was happy with my score but since most of my friends were re-appearing for the JEE Main, and that the best of the two would have been counted, so I sat for it again. I found the April exam to be easier than the January attempt," said Srivastava.

The Delhi boy hails from a family of engineers. His father works with the Air India and is an alumnus of Indian Institute of Technology (IIT)-Madras. His mother is a homemaker and his sister is studying B Tech from the Indraprastha Institute of Information Technology (IIIT)-Delhi. Srivastava too wishes to become an engineer. "I want to pursue computer science engineering from IIT-Delhi or IIT-Bombay. I love studying computers

because it has an application in real life. It can impact people's lives. It also involves Mathematics, networking which are my favourite areas of study," he said.

Srivastava started his JEE Main preparation right after class 10. "I was regular to school. I also joined a coaching class to get study material and training. Since the exam is based on NCERT books, mostly, the concepts are same and one can prepare for boards and entrance exam simultaneously," he said.

He referred to the NCERT books for Mathematics and Chemistry and for Physics, he focused on HC Verma. "What worked for me was my daily routine. After school and coaching, I devoted 3-4 hours to self-study daily. I kept my routine flexible otherwise. It is not important to refer to too many books but is important to get the concepts correct," said Srivastava who has started preparing for JEE Advanced already.

When asked how he maintained a routine for two years, he said, "I never got distracted by phone or games because they do not interest me. Solving problems gave me joy instead. In my free time, I used to read thriller suspense novels and play chess," said Srivastava who is a fan of Sherlock Holmes series.

Srivastava suggests his peers to take help of mock tests available on the NTA website. "It is important to refer to previous years questions to understand the type of questions that are asked in the exam," he said.

A total of 11,47,125 candidates appeared for JEE Main 2019-including both the January and April-out of which only 24 students secured 100 percentile.

Courtesy : The Indian Express

JEE 2019 MAIN

Hold on
8th April
(2nd Shift)

SOLVED PAPER

- If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is
(a) $3x - 2y = 0$ (b) $x - 2y + 8 = 0$
(c) $2x - y - 2 = 0$ (d) $2x - 3y + 10 = 0$
- Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x . Then $|\vec{a} \times \vec{b}| = r$ is possible if
(a) $r \geq 5\sqrt{\frac{3}{2}}$ (b) $\sqrt{\frac{3}{2}} < r \leq 3\sqrt{\frac{3}{2}}$
(c) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ (d) $0 < r \leq \sqrt{\frac{3}{2}}$
- If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is
(a) 4 : 5 : 6 (b) 5 : 9 : 13
(c) 3 : 4 : 5 (d) 5 : 6 : 7
- In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at (0, $5\sqrt{3}$), then the length of its latus rectum is
(a) 10 (b) 8 (c) 5 (d) 6
- If the system of linear equations
 $x - 2y + kz = 1$
 $2x + y + z = 2$
 $3x - y - kz = 3$
has a solution (x, y, z), $z \neq 0$, then (x, y) lies on the straight line whose equation is
(a) $3x - 4y - 1 = 0$ (b) $4x - 3y - 4 = 0$
(c) $4x - 3y - 1 = 0$ (d) $3x - 4y - 4 = 0$
- Let $f(x) = a^x$ ($a > 0$) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals
(a) $2f_1(x)f_1(y)$ (b) $2f_1(x+y)f_2(x-y)$
(c) $2f_1(x)f_2(y)$ (d) $2f_1(x+y)f_1(x-y)$
- Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function satisfying $f'(3) + f'(2) = 0$. Then $\lim_{x \rightarrow 0} \left(\frac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)} \right)^{1/x}$ is equal to
(a) 1 (b) e^{-1} (c) e (d) e^2
- Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $2y/x^2$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is
(a) $x \log_e |y| = x - 1$
(b) $x \log_e |y| = -2(x - 1)$
(c) $x \log_e |y| = 2(x - 1)$
(d) $x^2 \log_e |y| = -2(x - 1)$
- Let $S(\alpha) = \{(x, y) : y^2 \leq x, 0 \leq x \leq \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ , $0 < \lambda < 4$, $A(\lambda) : A(4) = 2 : 5$, then λ equals
(a) $2\left(\frac{2}{5}\right)^{1/3}$ (b) $4\left(\frac{4}{25}\right)^{1/3}$
(c) $4\left(\frac{2}{5}\right)^{1/3}$ (d) $2\left(\frac{4}{25}\right)^{1/3}$
- If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{1/3} + C$ where C is a constant of integration, then the function $f(x)$ is equal to
(a) $-\frac{1}{2x^3}$ (b) $\frac{3}{x^2}$ (c) $-\frac{1}{6x^3}$ (d) $-\frac{1}{2x^2}$
- A student scores the following marks in five tests : 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is
(a) $\frac{100}{\sqrt{3}}$ (b) $\frac{100}{3}$ (c) $\frac{10}{\sqrt{3}}$ (d) $\frac{10}{3}$



12. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5) = g(x)$, then $\int_0^x f(t)dt$ equals

(a) $\int_5^{x+5} g(t)dt$ (b) $\int_{x+5}^5 g(t)dt$
 (c) $5 \int_{x+5}^5 g(t)dt$ (d) $2 \int_5^{x+5} g(t)dt$

13. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?

(a) d, e, f , are in A.P. (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
 (c) d, e, f , are in G.P. (d) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

14. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) $\frac{2}{3}\sqrt{3}$ (d) $\sqrt{6}$

15. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is

(a) 3 (b) 5 (c) 2 (d) 4

16. The tangent and the normal lines at the point $(\sqrt{3}, 1)$ to the circle $x^2 + y^2 = 4$ and the x -axis form a triangle. The area of this triangle (in square units) is

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

17. Let the numbers 2, b, c be in an A.P. and

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$$

If $\det(A) \in [2, 16]$, then c lies in the interval

(a) $[2, 3]$ (b) $[4, 6]$
 (c) $[3, 2 + 2^{3/4}]$ (d) $(2 + 2^{3/4}, 4)$

18. If the fourth term in the binomial expansion

of $\left(\sqrt{x^{1+\log_{10} x}} + \frac{1}{x^{1/2}}\right)^6$ is equal to 200, and $x > 1$, then the value of x is
 (a) 100 (b) 10 (c) 10^3 (d) 10^4

19. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0, 1, 2, 3, 4, 5 (repetition of digits is allowed) is
 (a) 306 (b) 360 (c) 310 (d) 288

20. Which one of the following statements is not a tautology?

(a) $(p \vee q) \rightarrow (p \vee (\neg q))$
 (b) $p \rightarrow (p \vee q)$
 (c) $(p \wedge q) \rightarrow ((\neg p) \vee q)$
 (d) $(p \wedge q) \rightarrow p$

21. The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to

(a) $1 - \frac{11}{2^{20}}$ (b) $2 - \frac{21}{2^{20}}$
 (c) $2 - \frac{11}{2^{19}}$ (d) $2 - \frac{3}{2^{17}}$

22. Suppose that the points (h, k) , $(1, 2)$ and $(-3, 4)$ lie on the line L_1 . If a line L_2 passing through the points (h, k) and $(4, 3)$ is perpendicular to L_1 , then k/h equals

(a) $1/3$ (b) 0 (c) 3 (d) $-1/7$

23. The vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$ is

(a) $\vec{r} \times (\hat{i} + \hat{k}) + 2 = 0$ (b) $\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$
 (c) $\vec{r} \cdot (\hat{i} - \hat{k}) - 2 = 0$ (d) $\vec{r} \times (\hat{i} - \hat{k}) + 2 = 0$

24. Let $f: [-1, 3] \rightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \leq x < 1 \\ x + [x], & 1 \leq x < 2 \\ x + [x], & 2 \leq x \leq 3 \end{cases}$$

where $[t]$ denotes the greatest integer less than or equal to t . Then, f is discontinuous at

(a) only two points (b) only three points
 (c) four or more points (d) only one point

25. The number of integral values of m for which the equation $(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0$ has no real root is

(a) 3 (b) infinitely many
 (c) 2 (d) 1

26. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is

(a) 18 (b) 12 (c) 15 (d) 16





27. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2} (i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to

(a) $(-1 + 2i)^9$ (b) 0
(c) 1 (d) -1

28. If $f(1) = 1$, $f'(1) = 3$, then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is

(a) 12 (b) 9 (c) 15 (d) 33

29. The tangent to the parabola $y^2 = 4x$ at the point where it intersects the circle $x^2 + y^2 = 5$ in the first quadrant, passes through the point

(a) $(\frac{1}{4}, \frac{3}{4})$ (b) $(\frac{3}{4}, \frac{7}{4})$
(c) $(-\frac{1}{4}, \frac{1}{2})$ (d) $(-\frac{1}{3}, \frac{4}{3})$

30. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$, then the distance of R from the origin is

(a) 6 (b) $2\sqrt{21}$ (c) $\sqrt{53}$ (d) $2\sqrt{14}$

SOLUTIONS

1. (c) : Standard equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{Also, } e^2 = \left(1 + \frac{b^2}{a^2}\right) \Rightarrow 2^2 - 1 = \frac{b^2}{a^2}$$

[Given, eccentricity = 2]

$$\Rightarrow b^2 = 3a^2$$

Since, hyperbola passes through the point (4, 6).

$$\therefore \frac{16}{a^2} - \frac{36}{b^2} = 1 \Rightarrow \frac{16 \times 3 - 36}{3a^2} = 1 \quad [\because b^2 = 3a^2]$$

$$\Rightarrow 3a^2 = 12 \therefore a^2 = 4 \Rightarrow b^2 = 12$$

Now, equation of the tangent to the hyperbola at (4, 6) is

$$\frac{x \cdot 4}{4} - \frac{y \cdot 6}{12} = 1 \Rightarrow x - \frac{y}{2} = 1 \Rightarrow 2x - y - 2 = 0$$

2. (a) : Given $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(2+x) - \hat{j}(3-x) + \hat{k}(-3-2) = (2+x)\hat{i} + (x-3)\hat{j} - 5\hat{k}$$

$$\text{Now, } |\vec{a} \times \vec{b}| = \sqrt{(2+x)^2 + (x-3)^2 + (-5)^2}$$

$$= \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25} = \sqrt{2x^2 - 2x + 38}$$

$$\text{Let } f(x) = 2x^2 - 2x + 38 \Rightarrow f'(x) = 4x - 2 = 2(2x - 1)$$

$$\text{Put } f'(x) = 0 \Rightarrow x = \frac{1}{2}$$

$$\text{Also, } f''(x) = 4 > 0 \forall x.$$

$$\text{So, } f(x) \text{ is minimum for } x = \frac{1}{2}.$$

$$\therefore |\vec{a} \times \vec{b}| \text{ is minimum for } x = \frac{1}{2}.$$

$$\text{Thus, } |\vec{a} \times \vec{b}| \geq \sqrt{2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 38} = \sqrt{\frac{75}{2}} = 5\sqrt{\frac{3}{2}}$$

3. (a) : Let a, b, c be the lengths of the sides of the triangle ABC .

$$\text{Let } a < b < c \Rightarrow \angle C = 2\angle A \quad (\text{Given})$$

$$\Rightarrow \sin C = \sin 2A$$

$$\Rightarrow \sin C = 2 \sin A \cos A$$

$$\Rightarrow \frac{\sin C}{\sin A} = 2 \cos A$$

$$\Rightarrow \frac{c}{a} = 2 \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

[Using sine and cosine formula]

$$\Rightarrow bc^2 = a(b^2 + c^2 - a^2) \quad \dots(i)$$

Putting $a = b - \lambda$ and $c = b + \lambda, \lambda > 0$ in (i), we get

$$b(b + \lambda)^2 = (b - \lambda) [b^2 + (b + \lambda)^2 - (b - \lambda)^2]$$

[$\because a, b, c$ are in A.P.]

$$\Rightarrow b(b^2 + \lambda^2 + 2b\lambda) = (b - \lambda) (b^2 + 4b\lambda)$$

$$\Rightarrow b^2 + \lambda^2 + 2b\lambda = b^2 + 3b\lambda - 4\lambda^2 \Rightarrow 5\lambda^2 = b\lambda$$

$$\Rightarrow \lambda = \frac{b}{5} \therefore a = b - \frac{b}{5} = \frac{4b}{5} \text{ and } c = b + \frac{b}{5} = \frac{6b}{5}$$

Hence, the required ratio is 4 : 5 : 6.

4. (c) : Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

$$\text{Now, } 2a - 2b = 10 \Rightarrow a - b = 5 \quad \dots(i)$$

$$\text{Also, } ae = 5\sqrt{3} \quad \dots(ii)$$

$$\text{Now, } a^2 e^2 = a^2 - b^2 \Rightarrow (5\sqrt{3})^2 = a^2 - b^2 \quad (\text{Using (ii)})$$

$$\Rightarrow (a + b)(a - b) = 75$$

$$\Rightarrow a + b = 15 \quad \dots(iii) \text{ (Using (i))}$$

Solving (i) and (iii), we get

$$a = 10, b = 5$$

$$\therefore \text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times (5)^2}{10} = \frac{50}{10} = 5$$

5. (b) : The given system of linear equations is

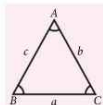
$$x - 2y + kz = 1 \quad \dots(i)$$

$$2x + y + z = 2 \quad \dots(ii)$$

$$3x - y - kz = 3 \quad \dots(iii)$$

Adding (i) and (iii), we get

$$4x - 3y - 4 = 0, \text{ which is the required straight line.}$$





6. (a) : Given, $f(x) = a^x$, $a > 0$

$$\Rightarrow f(x) = \frac{2a^x}{2} = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$$

$$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2} \text{ and } f_2(x) = \frac{a^x - a^{-x}}{2}$$

Consider $f_1(x+y) + f_1(x-y)$

$$\begin{aligned} &= \frac{a^{x+y} + a^{-(x+y)}}{2} + \frac{a^{x-y} + a^{-(x-y)}}{2} \\ &= \frac{a^{x+y}}{2} + \frac{a^{-x-y}}{2} + \frac{a^{x-y}}{2} + \frac{a^{-x+y}}{2} \\ &= a^y \left(\frac{a^x + a^{-x}}{2} \right) + a^{-y} \left(\frac{a^x + a^{-x}}{2} \right) \\ &= \left(\frac{a^x + a^{-x}}{2} \right) (a^y + a^{-y}) = f_1(x) \times 2f_1(y) = 2f_1(x)f_1(y) \end{aligned}$$

7. (a) : Consider,

$$\lim_{x \rightarrow 0} \left(\frac{1 + f(3+x) - f(3)}{1 + f(2-x) - f(2)} \right)^{\frac{1}{x}} \quad (1^\infty \text{ form})$$

$$\begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{1 + f(3+x) - f(3) - f(2-x) + f(2)}{1 + f(2-x) - f(2)} \right)^{\frac{1}{x}} \\ &= e^{\lim_{x \rightarrow 0} \frac{f(3+x) - f(3) - f(2-x) + f(2)}{x[1 + f(2-x) - f(2)]}} \\ &= e^{\lim_{x \rightarrow 0} \frac{f(3+x) - f(3)}{x-1} \cdot \lim_{x \rightarrow 0} \frac{f(2) - f(2-x)}{x-1}} \\ &= e^{f'(3) \cdot e^{f'(2)}} = e^{f'(3)+f'(2)} \\ &= e^0 \quad [\because f'(3) + f'(2) = 0] \\ &= 1 \end{aligned}$$

8. (c) : The given equation of circle is

$$x^2 + y^2 - 2x - 2y = 0$$

$$\therefore \text{Centre} \equiv (1, 1)$$

$$\text{Now, } \frac{dy}{dx} = \frac{2y}{x^2} \Rightarrow \int \frac{dy}{2y} = \int \frac{dx}{x^2} \quad [\text{Given}]$$

$$\Rightarrow \frac{1}{2} \log_e |y| = -\frac{1}{x} + c \quad \dots(i)$$

Since, the curve passes through (1, 1)

$$\therefore \frac{1}{2} \log_e 1 = -1 + c \Rightarrow c = 1$$

$$\text{From (i), } \frac{1}{2} \log_e |y| = -\frac{1}{x} + 1$$

$$\Rightarrow \frac{1}{2} \log_e |y| = \frac{x-1}{x} \Rightarrow x \log_e |y| = 2(x-1)$$

9. (b) : Here $S(\alpha) = \{(x, y) : y^2 \leq x, 0 \leq x \leq \alpha\}$ and $A(\alpha)$ is the area of region $S(\alpha)$.

$$\therefore A(\alpha) = 2 \int_0^\alpha \sqrt{x} \, dx = \frac{4}{3} (\alpha)^{3/2}$$

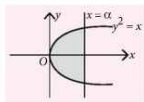
$$\text{Now, } A(\lambda) = \frac{4}{3} (\lambda)^{3/2}$$

$$\text{and } A(4) = \frac{4}{3} (4)^{3/2}$$

$$\text{Also, } \frac{A(\lambda)}{A(4)} = \frac{2}{5} \quad [\text{Given}]$$

$$\Rightarrow \frac{\frac{4}{3} (\lambda)^{3/2}}{\frac{4}{3} (4)^{3/2}} = \frac{2}{5} \Rightarrow \left(\frac{\lambda}{4} \right)^{3/2} = \frac{2}{5}$$

$$\Rightarrow \left(\frac{\lambda}{4} \right)^3 = \frac{4}{25} \Rightarrow \lambda = 4 \left(\frac{4}{25} \right)^{1/3}$$



$$10. (a) : \text{Given, } \int \frac{dx}{x^3 (1+x^6)^{2/3}} = x f(x) (1+x^6)^{1/3} + C$$

$$\Rightarrow \int \frac{dx}{x^7 \left(\frac{1}{x^6} + 1 \right)^{2/3}} = x f(x) (1+x^6)^{1/3} + C \quad \dots(i)$$

$$\text{Put } \frac{1}{x^6} + 1 = t \Rightarrow \frac{-6}{x^7} dx = dt \Rightarrow \frac{dx}{x^7} = \frac{-1}{6} dt$$

$$\therefore \int \frac{dx}{x^7 \left(\frac{1}{x^6} + 1 \right)^{2/3}} = \frac{-1}{6} \int \frac{dt}{t^{2/3}} = \frac{-1}{6} \times 3(t)^{1/3} + C$$

$$= \frac{-1}{2} \left(\frac{1}{x^6} + 1 \right)^{1/3} + C = \frac{-1}{2} \left(\frac{x^6 + 1}{x^6} \right)^{1/3} + C$$

$$= \frac{-1}{2x^2} (x^6 + 1)^{1/3} + C = x \left(\frac{-1}{2x^3} \right) (1+x^6)^{1/3} + C \quad \dots(ii)$$

Comparing (i) and (ii), we get

$$f(x) = \frac{-1}{2x^3}$$

11. (c) : Let the student scores x marks in the sixth test.

$$\text{Now, mean } (\bar{x}) = \frac{45 + 54 + 41 + 57 + 43 + x}{6}$$

$$\Rightarrow 48 \times 6 = 240 + x \quad [\because \text{Mean} = 48 \text{ (Given)}]$$

$$\Rightarrow x = 288 - 240 = 48$$

$$\text{Variance } (\sigma^2) = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$= \frac{(45)^2 + (54)^2 + (41)^2 + (57)^2 + (43)^2 + (48)^2}{6} - (48)^2$$

$$= \frac{14024}{6} - 2304 = \frac{200}{6} = \frac{100}{3}$$

$$\therefore \text{Standard deviation } (\sigma) = 10/\sqrt{3}$$





12. (b): Given, $f(x) = \int_0^x g(t) dt$

$$\text{Now, } f(-x) = \int_0^{-x} g(t) dt$$

$$\text{Put } t = -u \Rightarrow dt = -du$$

$$\Rightarrow f(-x) = \int_0^{-x} g(-u)(-du) = -\int_0^x g(-u) du$$

$$= -\int_0^x g(u) du \quad [\because g \text{ is an even function}]$$

$$= -f(x)$$

$$\therefore f(-x) = -f(x) \text{ i.e., } f(x) \text{ is an odd function.}$$

$$\text{Also, } f(x+5) = g(x) \quad \dots(i) \text{ [Given]}$$

$$\Rightarrow f(5-x) = g(-x) = g(x) = f(5+x)$$

$$\therefore f(5-x) = f(5+x) \quad \dots(ii)$$

$$\text{Now, } \int_0^x f(t) dt = \int_{-5}^{x-5} f(5+u) du$$

$$[\text{Put } t = 5 + u \Rightarrow dt = du]$$

$$= \int_{-5}^{x-5} g(u) du \quad [\text{Using (i)}]$$

$$= \int_{-5}^{x-5} f'(u) du \quad [\because f(x) = \int_0^x g(t) dt \Rightarrow f'(x) = g(x)]$$

$$= [f(u)]_{-5}^{x-5} = f(x-5) - f(-5)$$

$$= -f(5-x) + f(5) \quad [\because f(x) \text{ is an odd function}]$$

$$= f(5) - f(5+x) \quad (\text{Using (ii)})$$

$$= \int_{x+5}^5 f'(t) dt = \int_{x+5}^5 g(t) dt$$

$$= \int_{x+5}^5 g(t) dt$$

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$$= \int_{x+5}^5 g(t) dt$$

$$= 2\pi r^3 \cos^2 \theta \sin \theta$$

$$\Rightarrow \frac{dV}{d\theta} = 2\pi r^3 (2\cos \theta (-\sin \theta) \sin \theta + \cos^2 \theta \cos \theta)$$

$$= 2\pi r^3 (-2\cos \theta \sin^2 \theta + \cos^3 \theta)$$

$$\text{For maximum or minimum volume, } \frac{dV}{d\theta} = 0$$

$$\Rightarrow \cos^3 \theta = 2\cos \theta \sin^2 \theta \Rightarrow \cos^2 \theta = 2\sin^2 \theta$$

$$\Rightarrow \tan^2 \theta = \frac{1}{2} \Rightarrow \tan \theta = \frac{1}{\sqrt{2}} \Rightarrow \sin \theta = \frac{1}{\sqrt{3}}$$

$$\therefore h = 2r \sin \theta = 2 \times 3 \times \frac{1}{\sqrt{3}} \quad [\because r = 3 \text{ (Given)}]$$

$$= 2\sqrt{3}$$

15. (d): Probability of getting a head when a fair coin is tossed = $\frac{1}{2}$

Let the fair coin is tossed n -times.

Now, according to question,

P (at least one head) $\geq 90\%$

$$\Rightarrow 1 - \left(\frac{1}{2}\right)^n \geq \frac{90}{100}$$

$$\Rightarrow \left(\frac{1}{2}\right)^n \leq 1 - \frac{90}{100} \Rightarrow \left(\frac{1}{2}\right)^n \leq 0.1 \Rightarrow n \geq 4$$

Thus, minimum number of tosses = 4.

13. (d): Given a, b, c are in G.P.

$$\text{Let } a = a, b = ar \text{ and } c = ar^2 \quad \dots(i)$$

$$\text{Now, consider } ax^2 + 2bx + c = 0$$

$$\therefore ax^2 + 2arx + ar^2 = 0 \quad [\text{Using (i)}]$$

$$\Rightarrow x^2 + 2rx + r^2 = 0 \Rightarrow (x+r)^2 = 0 \Rightarrow x = -r$$

$$\text{Since, } x = -r \text{ is also a root of } dx^2 + 2ex + f = 0$$

$$\therefore dr^2 - 2er + f = 0$$

$$\Rightarrow d\left(\frac{c}{a}\right) - 2e\left(\frac{c}{b}\right) + f = 0$$

$$\left[\because (-r)(-r) = \frac{c}{a} \text{ and } \frac{ar^2}{ar} = \frac{c}{b} \right]$$

$$\Rightarrow \frac{dc}{a} + f = \frac{2ec}{b} \Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}$$

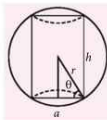
$$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$

14. (b): Let h be the height, a be the diameter of base of the cylinder and r be the radius of sphere.

$$\text{Now, } h = 2r \sin \theta, a = 2r \cos \theta$$

Volume of cylinder,

$$V = \pi (r \cos \theta)^2 (2r \sin \theta)$$



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16. (d) : Given equation of circle is $x^2 + y^2 = 4$... (i)

∴ Equation of tangent at $(\sqrt{3}, 1)$ is $\sqrt{3}x + y = 4$... (ii)

and equation of normal at

$(\sqrt{3}, 1)$ is $x - \sqrt{3}y = 0$

Putting $y = 0$ in (ii),

we get $x = \frac{4}{\sqrt{3}}$

∴ Co-ordinates of T are

$$\left(\frac{4}{\sqrt{3}}, 0\right)$$

Thus, area of triangle OPT = $\frac{1}{2} \times \frac{4}{\sqrt{3}} \times 1 = \frac{2}{\sqrt{3}}$

17. (b) : Given, 2, b, c are in A.P.

∴ $b = \frac{2+c}{2}$... (i)

$$\text{Also, } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$$

$$\begin{aligned} \therefore |A| &= 1(bc^2 - b^2c) - 1(2c^2 - 4c) + 1(2b^2 - 4b) \\ &= bc(c-b) - 2c^2 + 4c + 2b^2 - 4b \\ &= bc(c-b) - 2(c^2 - b^2) + 4(c-b) \\ &= (c-b)[bc - 2(c+b) + 4] = (c-b)[4 - 2c - 2b + bc] \\ &= (c-b)[2(2-c) - b(2-c)] = (c-b)(2-c)(2-b) \\ &= (c-b) \left(\frac{2+c}{2}\right) (2-c) \left(2 - \frac{2+c}{2}\right) = \left(\frac{c-2}{2}\right) (2-c) \left(\frac{2-c}{2}\right) \\ &= \frac{(c-2)^3}{4} \end{aligned}$$

Given, $|A| \in [2, 16]$

$$\therefore \frac{(c-2)^3}{4} \in [2, 16] \Rightarrow (c-2)^3 \in [8, 64]$$

$$\Rightarrow c-2 \in [2, 4] \Rightarrow c \in [4, 6]$$

18. (b) : Given, that $T_4 = 200$

$$\therefore {}^6C_3 \left(\frac{1}{x^{12}}\right)^3 \left(x^{\frac{1}{1+\log_{10} x}}\right)^2 = 200$$

$$\Rightarrow \frac{1}{x^4} \cdot x^{\frac{3}{2(1+\log_{10} x)}} = 10 \Rightarrow \frac{3}{2(1+\log_{10} x)} = 4 = 10$$

$$\text{Put } \log_{10} x = t \Rightarrow x = 10^t$$

$$\therefore 10^{\left[\frac{3}{2(1+t)} - 4\right]} = 10 \Rightarrow \left(\frac{3}{2(1+t)} + \frac{1}{4}\right) = 1$$

$$\Rightarrow (6+1+t)t = 4(1+t) \Rightarrow 7t + t^2 = 4 + 4t$$

$$\Rightarrow t^2 + 3t - 4 = 0 \Rightarrow (t-1)(t+4) = 0 \Rightarrow t = 1, -4$$

$$\Rightarrow x = 10, 10^{-4} \Rightarrow x = 10 \quad [\because x > 1]$$

Note : The expression $\left(\sqrt{x^{\frac{1}{1+\log_{10} x}} + x^{1/12}}\right)^6$ is wrongly

printed, it should be $\left(\sqrt{x^{\frac{1}{1+\log_{10} x}} + x^{\frac{1}{12}}}\right)^6$.

19. (c) : Number of four digit number starting with 432, 433, 434 and 435 = $4 + 6 + 6 + 6 = 22$

Number of four digit number starting with 44 and 45 = $6^2 + 6^2 = 36 + 36 = 72$

Number of four digit number starting with 5 = $6^3 = 216$

∴ Total numbers are = $22 + 72 + 216 = 310$

20. (a) :

p	q	$\neg p$	$\neg q$	$p \vee q$	$p \wedge q$	$p \vee (\neg q)$	$(\neg p) \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \vee q)$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	F	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	F	F	T	T
F	T	T	F	T	F	F	T	T	T	T
F	F	T	T	F	F	F	F	T	T	T

21. (c) : Consider $S = \sum_{k=1}^{20} k \cdot \frac{1}{2^k} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{20}{2^{20}}$... (i)

$$\Rightarrow S \times \frac{1}{2} = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{20}{2^{21}} \quad \dots (ii)$$

Subtracting (ii) from (i), we get

$$S \left(1 - \frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{20}} - \frac{20}{2^{21}}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2} \left[1 + \frac{1}{2} + \dots + \frac{1}{2^{19}} - \frac{20}{2^{20}}\right]$$

$$\Rightarrow S = 1 \left(\frac{1 - (1/2)^{20}}{1 - 1/2}\right) \cdot \frac{20}{2^{20}} \Rightarrow S = \frac{2^{20} - 1}{2^{20}} \cdot \frac{20}{2^{20}}$$

$$\Rightarrow S = \frac{2^{20} - 1 - 10}{2^{19}} = \frac{2^{20} - 11}{2^{19}} = 2 - \frac{11}{2^{19}}$$

22. (a) : Line L_1 passes through (1, 2) and (-3, 4).

∴ Equation of line L_1 is

$$y - 4 = \left(\frac{4-2}{-3-1}\right)(x+3)$$

$$\Rightarrow y - 4 = -\frac{1}{2}(x+3) \Rightarrow y = \frac{-x}{2} + \frac{5}{2} \quad \dots (i)$$

Now, slope of line L_2 is 2. [∵ $L_2 \perp L_1$]

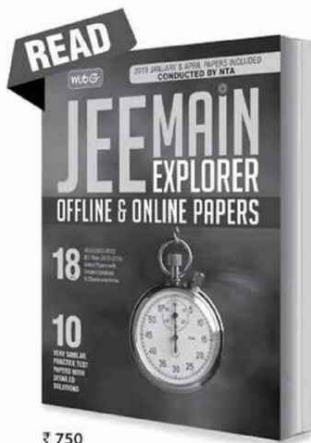
∴ Equation of line L_2 passing through (4, 3) is

$$y - 3 = 2(x - 4) \Rightarrow y = 2x - 5 \quad \dots (ii)$$

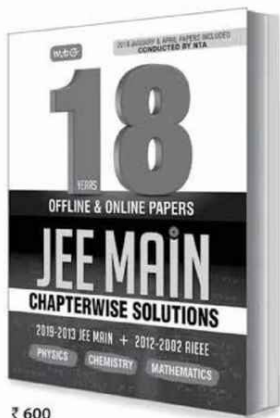


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Solving (i) and (ii), we get $x = 3$ and $y = 1$

$$\Rightarrow h = 3 \text{ and } k = 1 \quad \therefore \frac{k}{h} = \frac{1}{3}$$

23. (b): The equation of plane is

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0$$

$$\Rightarrow (2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z - 5\lambda - 1 = 0 \quad \dots(i)$$

Now, (i) is perpendicular to plane $x - y + z = 0$

$$\therefore (2\lambda + 1) - (3\lambda + 1) + (4\lambda + 1) = 0$$

$$\Rightarrow 3\lambda + 1 = 0 \Rightarrow \lambda = -1/3$$

$\therefore$ The equation of plane is $x - z + 2 = 0$

Hence, the equation of plane in vector form is

$$\vec{r} \cdot (\hat{i} - \hat{k}) + 2 = 0$$

$$\mathbf{24. (b):} \text{ Given } f(x) = \begin{cases} |x| + [x], & -1 \leq x < 1 \\ x + [x], & 1 \leq x < 2 \\ x + [x], & 2 \leq x \leq 3 \end{cases}$$

$$= \begin{cases} -x - 1, & -1 \leq x < 0 \\ x, & 0 \leq x < 1 \\ 2x, & 1 \leq x < 2 \\ x + 2, & 2 \leq x < 3 \\ x + 3, & x = 3 \end{cases}$$

Thus, f is discontinuous at $x = 0, 1, 3$.

25. (b): Since the given equation

$$(1 + m^2)x^2 - 2(1 + 3m)x + (1 + 8m) = 0 \text{ has no real root.}$$

$$\therefore D < 0$$

$$\Rightarrow 4(1 + 3m)^2 - 4(1 + m^2)(1 + 8m) < 0$$

$$\Rightarrow (1 + 3m)^2 < (1 + m^2)(1 + 8m)$$

$$\Rightarrow 1 + 9m^2 + 6m < 1 + 8m + m^2 + 8m^3$$

$$\Rightarrow 8m^3 - 8m^2 + 2m > 0 \Rightarrow 2m(4m^2 - 4m + 1) > 0$$

$$\Rightarrow 2m(2m - 1)^2 > 0 \Rightarrow m > 0$$

$\therefore$ There exist infinitely many integral values of m .

26. (d): Let AB and CD be two vertical poles of height 20 m and 80 m. Also, $BF = x_1$ and $FD = x_2$.

Now, $\triangle EBF \sim \triangle CBD$

$$\therefore \frac{h}{x_1} = \frac{80}{x_1 + x_2} \quad \dots(i)$$

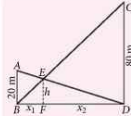
Similarly,

$$\triangle EFD \sim \triangle ABD \Rightarrow \frac{h}{x_2} = \frac{20}{x_1 + x_2}$$

$$\text{From (i) and (ii), } 80x_1 = 20x_2$$

$$\Rightarrow \frac{x_2}{x_1} = 4 \Rightarrow x_2 = 4x_1$$

$$\text{From (i), } \frac{h}{x_1} = \frac{80}{5x_1} \Rightarrow h = 16 \text{ m}$$



$\dots(ii)$

$$\mathbf{27. (d):} \text{ Given, } z = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$\text{Now, } z^5 = \cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} = -\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$$

$$= \frac{-\sqrt{3}}{2} + \frac{i}{2} = \frac{-\sqrt{3} + i}{2}$$

$$\text{Also, } z^8 = \cos \frac{8\pi}{6} + i \sin \frac{8\pi}{6} = \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$$

$$= -\cos \frac{\pi}{3} - i \sin \frac{\pi}{3} = -\frac{1}{2} - \frac{i\sqrt{3}}{2} = -\left(\frac{1 + i\sqrt{3}}{2}\right)$$

$$\text{Now, } (1 + iz + z^5 + iz^8)^9$$

$$= \left[1 + \left(\frac{\sqrt{3} + i}{2}\right) + \left(\frac{-\sqrt{3} + i}{2}\right) - i\left(\frac{1 + i\sqrt{3}}{2}\right) \right]^9$$

$$= \left[1 + \frac{i\sqrt{3}}{2} - \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{i}{2} - \frac{i}{2} + \frac{\sqrt{3}}{2} \right]^9$$

$$= \left(\frac{1 + i\sqrt{3}}{2}\right)^9 = \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)^9$$

$$= \cos 3\pi + i \sin 3\pi = -1$$

$$\mathbf{28. (d):} \text{ Let } y = f(f(f(x))) + (f(x))^2$$

$$\Rightarrow \frac{dy}{dx} = f'(f(f(x)))f'(f(x))f'(x) + 2f(x)f'(x)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=1} = f'(f(f(1)))f'(f(1))f'(1) + 2f(1)f'(1)$$

$$= f'(1)f'(1)f'(1) + 2f'(1) \quad [\because f(1) = 1]$$

$$= (3)^3 + 2 \times 3 = 27 + 6 = 33 \quad [\because f'(1) = 3]$$

29. (b): The given equation of parabola is $y^2 = 4x$

$\dots(i)$

$$\text{and circle is } x^2 + y^2 = 5$$

$\dots(ii)$

$$\text{From (i) and (ii), } x^2 + 4x - 5 = 0$$

$$\Rightarrow (x + 5)(x - 1) = 0 \Rightarrow x = 1 \text{ or } -5$$

$$\Rightarrow x = 1 \text{ and } y = 2 \quad [\because x \text{ and } y \text{ are in } 1^{\text{st}} \text{ quadrant}]$$

$$\therefore \text{Equation of tangent at } (1, 2) \text{ to the parabola}$$

$$y^2 = 4x \text{ is } y(2) = 2(x + 1) \Rightarrow y = x + 1$$

From the options, we see that point $(3/4, 7/4)$ given in

option (b) satisfies the equation of tangent.

30. (d): Given, $R(4, y, z)$ lies on the line segment joining the points $P(2, -3, 4)$ and $Q(8, 0, 10)$.

$$\therefore \frac{8-4}{4-2} = \frac{0-y}{y+3} = \frac{10-z}{z-4} \Rightarrow \frac{4}{2} = \frac{-y}{y+3} = \frac{10-z}{z-4}$$

$$\therefore y = -2 \text{ and } z = 6$$

$$\therefore \text{Distance of } R(4, -2, 6) \text{ from } O(0, 0, 0)$$

$$= \sqrt{16 + 4 + 36} = \sqrt{56} = 2\sqrt{14} \text{ units}$$





MOCK TEST PAPER 2020 JEE MAIN

Time : 1 hr 15 min.

Series - 1

The entire syllabus of Mathematics of JEE MAIN is being divided in to eight units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues. The syllabus for module break-up is given below:

	Topic	Syllabus In Detail
UNIT NO. 1	Sets, Relations & Functions	Sets and their representation; Union, intersection and complement of sets and their algebraic properties; Power set. Ordered pairs, Cartesian product of sets. Definition of relation. Domain and range of a relation. Types of relations, equivalence relations. Function as a special kind of relation from one set to another. Domain, co-domain and range of a function. One-one, into and onto functions, Composition of functions. Real valued function of a real variable, domain and range of these functions, constant, identity, polynomial, rational, modulus, Signum and greatest integer functions with their graphs, sum, difference, product and quotients of functions. Odd & even functions and periodic function.
	Trigonometry	Measurement of Trigonometric Angles. Definition of Trigonometric functions, Associated Angles
	Co-ordinate Geometry-2D	Cartesian coordinates, distance between two points, section formulae, shift of origin. Locus problems.

1. Let A and B be two sets such that $A \cap X = B \cap X = \emptyset$ and $A \cup X = B \cup X$ for some set X . Then,

(a) $A = B$ (b) $A = X$ (c) $B = X$ (d) $A \cup B = X$

2. Let A and B be two sets defined as given below :

$A = \{(x, y) : |x - 3| < 1 \text{ and } |y - 3| < 1\}$ and

$B = \{(x, y) : 4x^2 + 9y^2 - 32x - 54y + 109 \leq 0\}$. Then,

(a) $A \subset B$ (b) $B \subset A$
(c) $A = B$ (d) none of these

3. On the set N of natural numbers, define the relation R by aRb iff the G.C.D. of a and b is 2, then R is

(a) reflexive but not symmetric
(b) symmetric only
(c) equivalence
(d) neither reflexive nor symmetric and transitive.

4. The value of

$$\sin^6\left(\frac{\pi}{49}\right) + \cos^6\left(\frac{\pi}{49}\right) - 1 + 3\sin^2\left(\frac{\pi}{49}\right)\cos^2\left(\frac{\pi}{49}\right) =$$

(a) $\tan^6\left(\frac{\pi}{49}\right)$ (b) $\cot^6\left(\frac{\pi}{49}\right)$
(c) 1 (d) 0

5. If $\sec 2\theta = p + \tan 2\theta$, then the value of $\sin^2\theta$ in terms of p is given by

(a) $\frac{(p-1)^2}{2(p^2+1)}$

(b) $\frac{1}{2} \left(\frac{p-1}{p+1} \right)^2$

(c) $\frac{p^2-1}{2(p^2+1)}$

(d) $\frac{p^2-1}{2(p+1)^2}$

6. If $x \cos \alpha + y \sin \alpha = x \cos \beta + y \sin \beta = 2a$, then $\cos \alpha \cos \beta =$

(a) $\frac{4ax}{x^2+y^2}$

(b) $\frac{4a^2-y^2}{x^2+y^2}$

(c) $\frac{4ay}{x^2+y^2}$

(d) $\frac{4a^2-x^2}{x^2+y^2}$

7. One possible condition for the three points (a, b) , (b, a) and $(a^2, -b^2)$ to be collinear is

(a) $a - b = 2$ (b) $a + b = 2$
(c) $a = 1 + b$ (d) $a = 1 - b$

8. If the points $O(0, 0)$, $A(\cos \alpha, \sin \alpha)$, $B(\cos \beta, \sin \beta)$ are the vertices of a right angled triangle, then

$$\left| \sin \left(\frac{\alpha - \beta}{2} \right) \right| =$$

By : Sankar Ghosh, S.G.M.C. Mob. 09831244397.



- (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$
 (c) $\frac{1}{\sqrt{3}}$ (d) none of these

9. $OPQR$ is a square and M, N are the middle points of the sides PQ and QR respectively, then the ratio of the areas of the square and the triangle OMN , is

- (a) 4 : 1 (b) 2 : 1 (c) 8 : 3 (d) 4 : 3

10. Let $A(5, 12)$, $B(-13 \cos \theta, 13 \sin \theta)$ and $C(13 \sin \theta, -13 \cos \theta)$ be the vertices of $\triangle ABC$, where $\theta \in R$. Then the locus of the orthocentre of $\triangle ABC$ is

- (a) $x - y + 7 = 0$ (b) $x - y - 7 = 0$
 (c) $x + y - 7 = 0$ (d) $x + y + 7 = 0$

11. Area of the triangle with vertices (a, b) , (x_1, y_1) and (x_2, y_2) where a, x_1, x_2 are in G.P. with common ratio r and b, y_1, y_2 are in G.P. with common ratio s , is

- (a) $ab(r-1)(s-1)(s-r)$
 (b) $\frac{1}{2}ab(r+1)(s+1)(s-r)$
 (c) $\frac{1}{2}ab(r-1)(s-1)(s-r)$
 (d) $\frac{1}{2}ab(r+1)(s+1)(r-s)$

12. If $e^{(\sin^2 x + \sin^4 x + \sin^6 x + \dots \text{to } \infty) \log_e 2}$ satisfies the equation $x^2 - 9x + 8 = 0$, then the value of

- $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$ is
 (a) $\sqrt{3} + 1$ (b) $\frac{1}{2}(\sqrt{3} + 1)$
 (c) $\frac{1}{2}(\sqrt{3} - 1)$ (d) $\sqrt{3} - 1$

13. For any three sets A_1, A_2, A_3 , let $B_1 = A_1, B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then

- (a) $A_1 \cup A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$
 (b) $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$
 (c) $A_1 \cup A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$
 (d) none of these

14. If $f: A \rightarrow B$ given by $3^{f(x)} + 2^{-x} = 4$ is a bijection, then

- (a) $A = \{x \in R : -1 < x < \infty\}$, $B = \{x \in R : 2 < x < 4\}$
 (b) $A = \{x \in R : -3 < x < \infty\}$, $B = \{x \in R : 0 < x < 4\}$
 (c) $A = \{x \in R : -2 < x < \infty\}$, $B = \{x \in R : 0 < x < 4\}$
 (d) none of these

15. The period of the function $f(x) = |\cos 4x| + |\sin 4x|$ is

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

16. If the vertices of a triangle are $\left(4, \frac{1}{4}\right), \left(3, \frac{1}{3}\right), (1, 1)$, then orthocentre of the triangle is

- (a) $\left(\frac{1}{12}, 12\right)$ (b) $\left(12, \frac{1}{12}\right)$
 (c) $\left(-\frac{1}{12}, -12\right)$ (d) none of these

17. The function $f: R \rightarrow R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is

- (a) one-one but not onto (b) onto but not one-one
 (c) both one-one and onto
 (d) neither one-one nor onto.

18. $f: R \times R \rightarrow R$ such that $f(x+iy) = \sqrt{x^2+y^2}$ then f is

- (a) many-one into (b) one-one and onto
 (c) many-one and onto (d) one-one and into.

19. The range of the function $f(x) = \sin[x], \frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ where $[x]$ denotes the greatest integer $\leq x$ is

- (a) $\{0\}$ (b) $\{0, -1\}$
 (c) $\{0, \pm \sin 1\}$ (d) $\{-\sin 1, 0\}$

20. Let $f(x) = \frac{\sqrt{\sin x}}{1 + \sqrt{\sin x}}$. Then domain of f contains

- (a) $(0, \pi)$ (b) $(0, 2\pi)$ (c) $(2\pi, 4\pi)$ (d) $(4\pi, 6\pi)$

21. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right-angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which K can take is given by

- (a) $\{0, 2\}$ (b) $\{-1, 3\}$ (c) $\{-3, -2\}$ (d) $\{1, 3\}$

22. If the coordinate axes are shifted to the point $(-1, 2)$ without rotation, then the curve whose equation is $2x^2 + y^2 - 4x + 4y = 0$ will have the equation

- (a) $2x^2 + y^2 - 8x + 8y + 18 = 0$
 (b) $2x^2 + y^2 - 6 = 0$
 (c) $2x^2 + y^2 + 8y + 10 = 0$
 (d) $x^2 + 2y^2 + 8x - 8y + 18 = 0$

23. The function $f(x) = \sqrt{3} \sin 2x - \cos 2x + 4$ is one-one in the interval

- (a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
 (c) $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ (d) $[-\pi, \pi]$

24. If the real valued function $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is even, then n is equals to

- (a) 2 (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 3





25. If R_1 and R_2 are symmetric relations (not disjoint) on a set A , then the relation $R_1 \cap R_2$ is

- (a) reflexive (b) symmetric
(c) transitive (d) none of these.

26. The relation R defined in $A = \{1, 2, 3\}$ by aRb , if $|a^2 - b^2| \leq 5$. Which of the following is false?

- (a) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$
(b) $R^{-1} = R$
(c) Domain of $R = \{1, 2, 3\}$
(d) Range of $R = \{5\}$

27. Let $f(x) = (x+1)^2 - 1$ ($x \geq -1$), then the set $\{x : f^{-1}(x) = f(x)\}$ contains

- (a) $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$
(b) $\{0, -1, 1\}$
(c) $\{0, -1\}$ (d) $\{x : x(x-1) = 0\}$

28. If $x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3}$, then $xy + yz + zx =$

- (a) -1 (b) 0 (c) 1 (d) 2

29. Let $g(x) = Hx - [x]$, where $[x]$ is the greatest integer not greater than x .

If $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$, then for all x , $f(g(x))$ equals

- (a) x (b) 1 (c) $f(x)$ (d) $g(x)$

30. Let $f : (4, 6) \rightarrow (6, 8)$ be a function defined by

$$f(x) = x + \left[\frac{x}{2} \right] \text{ (where } [\cdot] \text{ denotes the greatest integer function), then } f^{-1}(x) \text{ equal to}$$

- (a) $x - \left[\frac{x}{2} \right]$ (b) $-x - 2$ (c) $x - 2$ (d) $\frac{1}{x + \left[\frac{\pi}{6} \right]}$

SOLUTIONS

1. (a) : We have $A \cup X = B \cup X$ for some set X

$$\Rightarrow A \cap (A \cup X) = A \cap (B \cup X)$$

$$\Rightarrow (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$$

$$\Rightarrow A \cup \phi = (A \cap B) \cup \phi \quad [\because A \cap X = \phi]$$

$$\Rightarrow A = (A \cap B) \quad \dots(i)$$

$$\text{Again, } A \cup X = B \cup X$$

$$\Rightarrow B \cap (A \cup X) = B \cap (B \cup X)$$

$$\Rightarrow (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$$

$$\Rightarrow (B \cap A) \cup \phi = B \cup \phi \quad [\because B \cap X = \phi]$$

$$\Rightarrow B = (A \cap B) \quad \dots(ii)$$

From (i) and (ii), we have $A = B$.

2. (a) : We have the sets $A = \{(x, y) : |x-3| < 1 \text{ and } |y-3| < 1\}$ and $B = \{(x, y) : 4x^2 + 9y^2 - 32x - 54y + 109 \leq 0\}$

$$\therefore |x-3| < 1 \text{ and } |y-3| < 1$$

$$\Rightarrow -1 < (x-3) < 1 \text{ and } -1 < (y-3) < 1$$

$$\Rightarrow 2 < x < 4 \text{ and } 2 < y < 4$$

Thus, A is the set of all points (x, y) lying inside the square formed by the lines $x = 2, x = 4, y = 2$ and $y = 4$.

$$\text{Now, } 4(x^2 - 8x) + 9(y^2 - 6y) + 109 \leq 0$$

$$\Rightarrow 4(x-4)^2 + 9(y-3)^2 \leq 36$$

$$\Rightarrow \frac{(x-4)^2}{9} + \frac{(y-3)^2}{4} \leq 1$$

Thus, B is the set of all points lying inside the ellipse having its centre at $(4, 3)$ and major and minor axes of length 3 and 2 unit respectively.

It can be easily seen from the figure that $A \subset B$.

3. (b) : For any $a \in N$, we have G.C.D. $(a, a) = a$.

Thus R is not reflexive.

Let $(a, b) \in R$. Then aRb

$\Rightarrow$ G.C.D. $(a, b) = 2 =$ G.C.D. $(b, a) \Rightarrow bRa$. So, R is symmetric.

We observed that G.C.D. $(6, 4) = 2$ and G.C.D. $(4, 18) = 2$. But, G.C.D. $(6, 18) = 6$.

Hence, $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$ is not satisfied. So, R is not transitive.

$$\begin{aligned} 4. (d) : \sin^6\left(\frac{\pi}{49}\right) + \cos^6\left(\frac{\pi}{49}\right) - 1 + 3\sin^2\left(\frac{\pi}{49}\right)\cos^2\left(\frac{\pi}{49}\right) \\ = \sin^6\left(\frac{\pi}{49}\right) + \cos^6\left(\frac{\pi}{49}\right) \\ + 3\sin^2\left(\frac{\pi}{49}\right)\cos^2\left(\frac{\pi}{49}\right)\left(\sin^2\left(\frac{\pi}{49}\right) + \cos^2\left(\frac{\pi}{49}\right)\right) - 1 \\ = \left(\sin^2\left(\frac{\pi}{49}\right) + \cos^2\left(\frac{\pi}{49}\right)\right)^3 - 1 = 1 - 1 = 0. \end{aligned}$$

5. (a) : We have, $\sec 2\theta = p + \tan 2\theta$

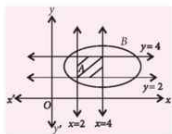
$$\Rightarrow \sec 2\theta - \tan 2\theta = p \quad \dots(i)$$

$$\Rightarrow \sec 2\theta + \tan 2\theta = \frac{1}{p} \quad \dots(ii)$$

$$[\because \sec^2 2\theta - \tan^2 2\theta = 1]$$

Adding (i) and (ii), we get $2\sec 2\theta = p + \frac{1}{p}$

$$\Rightarrow \cos 2\theta = \frac{2p}{p^2 + 1} \Rightarrow 1 - 2\sin^2 \theta = \frac{2p}{p^2 + 1}$$





$$\Rightarrow 2\sin^2\theta = \frac{(p-1)^2}{p^2+1} \Rightarrow \sin^2\theta = \frac{(p-1)^2}{2(p^2+1)}$$

6. (b): Given, $x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta = 2a$

$\Rightarrow \alpha$ and β are the roots of $x\cos\theta + y\sin\theta = 2a$

Now, $x\cos\theta + y\sin\theta = 2a$

$$\Rightarrow (x\cos\theta - 2a)^2 = y^2\sin^2\theta$$

$$\Rightarrow x^2\cos^2\theta + 4a^2 - 4ax\cos\theta = y^2(1 - \cos^2\theta)$$

$$\Rightarrow (x^2 + y^2)\cos^2\theta - 4ax\cos\theta + 4a^2 - y^2 = 0$$

Clearly, $\cos\alpha, \cos\beta$ are the roots of this equation.

$$\therefore \cos\alpha\cos\beta = \frac{4a^2 - y^2}{x^2 + y^2}$$

7. (c): Given points will be collinear, if the area of the triangle formed by the points (a, b) , (b, a) and $(a^2, -b^2)$ will be zero.

$$\text{Therefore } \frac{1}{2}\{a(a+b^2) + b(-b^2 - b) + a^2(b-a)\} = 0$$

$$\Rightarrow a(a+b^2) + b(-b^2 - b) + a^2(b-a) = 0$$

$$\Rightarrow a^2 + ab^2 - b^3 - b^2 + a^2b - a^3 = 0$$

$$\Rightarrow a^2 - b^2 + b^2(a-b) - a^2(a-b) = 0$$

$$\Rightarrow (a-b)(a+b+b^2-a^2) = 0$$

$$\Rightarrow (a-b)(a+b)(1+b-a) = 0$$

$$\Rightarrow a = b \text{ or } a + b = 0 \text{ or } a = b + 1$$

8. (b): We have

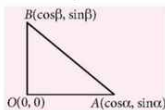
$$OA = \sqrt{(\cos\alpha - 0)^2 + (\sin\alpha - 0)^2} = \sqrt{\cos^2\alpha + \sin^2\alpha} = 1$$

$$OB = \sqrt{(\cos\beta - 0)^2 + (\sin\beta - 0)^2} = \sqrt{\cos^2\beta + \sin^2\beta} = 1$$

$$\text{and } AB = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}$$

$$= \sqrt{2 - 2\cos(\alpha - \beta)}$$

$$= 2 \left| \sin \left(\frac{\alpha - \beta}{2} \right) \right|$$



Since $\triangle OAB$ is a right triangle.

$$\therefore AB = \sqrt{OA^2 + OB^2}$$

$$\Rightarrow 2 \left| \sin \frac{\alpha - \beta}{2} \right| = \sqrt{2} \Rightarrow \left| \sin \frac{\alpha - \beta}{2} \right| = \frac{1}{\sqrt{2}}$$

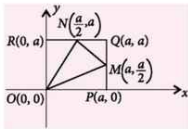
9. (c): Let $O(0, 0)$, $P(a, 0)$, $Q(a, a)$ and $R(0, a)$ are the vertices of the square $OPQR$. Therefore the coordinates

of M and N are respectively $\left(\frac{a}{2}, \frac{a}{2}\right)$ and $\left(\frac{a}{2}, a\right)$.

Now, area of $\triangle OMN$

$$= \frac{1}{2} \left| 0 \left(\frac{a}{2} - a \right) + a \left(a - 0 \right) + \frac{a}{2} \left(0 - \frac{a}{2} \right) \right|$$

$$= \frac{1}{2} \left(a^2 - \frac{a^2}{4} \right) \\ = \frac{1}{2} \times \frac{3}{4} a^2 \\ = \frac{3}{8} a^2$$



Area of the square $OPQR = a^2$

$\therefore$ Required ratio = $8 : 3$

10. (a): Let $O(0, 0)$ be the origin.

Then, $OA = OB = OC = 13$

$\therefore$ Origin is the circumcentre of $\triangle ABC$.

Let $O'(h, k)$ be the orthocentre of $\triangle ABC$ and G be the centroid.

The co-ordinates of G are

$$\left(\frac{5 - 13\cos\theta + 13\sin\theta}{3}, \frac{12 + 13\sin\theta - 13\cos\theta}{3} \right)$$

The points O, G and O' are collinear and G divides OO' in the ratio $1 : 2$

$$\therefore \frac{5 - 13\cos\theta + 13\sin\theta}{3} = \frac{h}{3}$$

$$\text{and } \frac{12 + 13\sin\theta - 13\cos\theta}{3} = \frac{k}{3}$$

$$\Rightarrow \sin\theta - \cos\theta = \frac{h-5}{13} \text{ and } \sin\theta - \cos\theta = \frac{k-12}{13}$$

$$\Rightarrow \frac{h-5}{13} = \frac{k-12}{13} \Rightarrow h-k+7=0$$

Hence the locus of (h, k) is $x - y + 7 = 0$.

11. (c): We have, $x_1 = ar$, $x_2 = ar^2$, $y_1 = bs$, $y_2 = bs^2$

$\therefore$ Area of the triangle is given by

$$\Delta = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \frac{1}{2} \begin{vmatrix} a & b & 1 \\ ar & bs & 1 \\ ar^2 & bs^2 & 1 \end{vmatrix} = \frac{1}{2} ab \begin{vmatrix} r & s & 1 \\ r^2 & s^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2} ab \begin{vmatrix} 1 & 1 & 1 \\ r-1 & s-1 & 0 \\ r^2-1 & s^2-1 & 0 \end{vmatrix}$$

$$[\text{Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1]$$

$$= \frac{1}{2} ab \begin{vmatrix} 0 & 0 & 1 \\ r-s & s-1 & 0 \\ r^2-s^2 & s^2-1 & 0 \end{vmatrix}$$

$$[\text{Applying } C_1 \rightarrow C_1 - C_2 \text{ and } C_2 \rightarrow C_2 - C_3]$$





$$= \frac{1}{2} ab(r-s)(s-1) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ r+s & s+1 & 0 \end{vmatrix}$$

$$= \frac{1}{2} ab(r-s)(s-1)(1-r) = \frac{1}{2} ab(r-1)(s-1)(s-r)$$

12. (c) : $e^{\left(\sin^2 x + \sin^4 x + \sin^6 x + \dots \text{to } \infty\right) \log_e 2}$

$$= e^{\left(\frac{\sin^2 x}{1 - \sin^2 x}\right) \log_e 2} = e^{\tan^2 x \log_e 2} = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$$

Now, $x^2 - 9x + 8 = 0 \Rightarrow x = 1$ or $x = 8$

$\therefore 2^{\tan^2 x} = 1$ or $8 \Rightarrow 2^{\tan^2 x} = 2^0$ or 2^3

Now, $\tan^2 x = 0 \Rightarrow x = 0$ or $\tan^2 x = 3 \Rightarrow x = \pm \frac{\pi}{3}$

But only $\frac{\pi}{3}$ belongs to $0 < x < \frac{\pi}{2}$. So, $x = \frac{\pi}{3}$.

$$\therefore \frac{\cos x}{\cos x + \sin x} = \frac{\cos \pi/3}{\cos \pi/3 + \sin \pi/3} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{\sqrt{3}-1}{2}$$

13. (b) : $B_1 \cup B_2 = A_1 \cup (A_2 - A_1) = A_1 \cup (A_2 \cap A_1')$
 $= (A_1 \cup A_2) \cap (A_1 \cup A_1') = (A_1 \cup A_2) \cap U = (A_1 \cup A_2)$
 Now, $B_1 \cup B_2 \cup B_3 = (B_1 \cup B_2) \cup B_3 = (A_1 \cup A_2) \cup [A_3 - (A_1 \cup A_2)]$
 $= (A_1 \cup A_2) \cup [A_3 \cap (A_1 \cup A_2)']$
 $= [(A_1 \cup A_2) \cup A_3] \cap [(A_1 \cup A_2) \cup (A_1 \cup A_2)']$
 $= [A_1 \cup A_2 \cup A_3] \cap U = A_1 \cup A_2 \cup A_3$.

14. (d) : We have, $3^{f(x)} + 2^{-x} = 4 \Rightarrow 3^{f(x)} = 4 - 2^{-x}$
 $\Rightarrow f(x) = \log_3(4 - 2^{-x})$
 Clearly, $f(x)$ will be defined, if $4 - 2^{-x} > 0$
 $\Rightarrow 2^{-x} < 4 \Rightarrow 2^x > 2^{-2} \Rightarrow x > -2$
 $\Rightarrow A = \{x \in \mathbb{R} : x > -2\}$
 Now, $f(x) = \log_3(4 - 2^{-x})$

$$\Rightarrow f'(x) = \frac{2^{-x} \log 2}{(4 - 2^{-x}) \log 3} > 0 \quad \forall x > -2$$

$\Rightarrow f(x)$ is increasing on $(-2, \infty)$
 $\Rightarrow$ Range of $f(x) = \{y : f(-2) < y < f(\infty)\}$
 $\Rightarrow B = \{y : -\infty < y < \log_3 4\} = \{x : -\infty < x < \log_3 4\}$

15. (d) : Here $f(x) = |\cos 4x| + |\sin 4x|$

$$\therefore f\left(x + \frac{\pi}{8}\right) = \left|\cos 4\left(x + \frac{\pi}{8}\right)\right| + \left|\sin 4\left(x + \frac{\pi}{8}\right)\right|$$

$$= |\sin 4x| + |\cos 4x| = f(x).$$

$\therefore f\left(x + \frac{\pi}{8}\right) = f(x)$, therefore the period of the given function is $\frac{\pi}{8}$.

16. (c) : We know that, if vertices of a $\triangle ABC$ are $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$, then the coordinate of orthocentre is $\left(-\frac{1}{abc}, -abc\right)$.

17. (b) : The given function is $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = (x-1)(x-2)(x-3)$
 Now, $1, 2, 3 \in \mathbb{R}$ and $f(1) = 0, f(2) = 0, f(3) = 0$.
 $\therefore f(1) = f(2)$ but $1 \neq 2$
 $\therefore f$ is not one-to-one mapping.
 Again, if $y = (x-1)(x-2)(x-3)$, then $x^3 - 6x^2 + 11x - 6 - y = 0$ and any third degree equation must have atleast one real root. Hence it is an onto mapping.

18. (a) : Since $f(x+iy) = f(x-iy) \therefore f$ is many one.
 Also range $\in \mathbb{R}^+$ and co-domain $\in \mathbb{R}$
 $\therefore$ range $\subset$ co-domain $\Rightarrow f$ is into
 Hence f is many-one into.

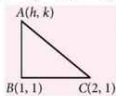
19. (d) : The given function is $f(x) = \sin[x]$
 Clearly $-\frac{\pi}{4} \leq x < 0 \Rightarrow [x] = -1$ and $0 \leq x < \frac{\pi}{4} \Rightarrow [x] = 0$
 $\therefore$ Range of $f = \{\sin(-1), \sin 0\} = \{-\sin 1, 0\}$

20. (a) : The given function is $f(x) = \frac{\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$
 $f(x)$ is defined when $\sin x > 0$ and $1 + \sqrt[3]{\sin x} \neq 0$

Clearly $x \in (0, \pi)$
 $\therefore$ Required domain = $(0, \pi)$.

21. (b) : We know that $AB^2 + BC^2 = AC^2$
 $\Rightarrow (h-1)^2 + (k-1)^2 + (2-1)^2 + (1-1)^2 = (h-2)^2 + (k-1)^2$
 $\Rightarrow 2h = 2 \Rightarrow h = 1$

Also, $\frac{1}{2} BC \cdot AB = 1$ [Given]
 $\Rightarrow |1(k-1)| = 2$
 $\Rightarrow k-1 = \pm 2 \Rightarrow k = 3, -1$



22. (a) : Here the transformation is $x = x' - 1$ and $y = y' + 2$
 The given equation becomes
 $2(x'-1)^2 + (y'+2)^2 - 4(x'-1) + 4(y'+2) = 0$
 $\Rightarrow 2x'^2 + y'^2 - 8x' + 8y' + 18 = 0$
 or $2x'^2 + y'^2 - 8x' + 8y' + 18 = 0$

23. (c) : Here, $f(x) = \sqrt{3} \sin 2x - \cos 2x + 4$
 $\Rightarrow f(x) = 2\left(\frac{\sqrt{3}}{2} \sin 2x - \frac{1}{2} \cos 2x\right) + 4$



$$\Rightarrow f(x) = 2 \sin \left(2x - \frac{\pi}{6} \right) + 4$$

We know that $\sin x$ is one-one in $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

$$\therefore -\frac{\pi}{2} \leq 2x - \frac{\pi}{6} \leq \frac{\pi}{2} \Rightarrow -\frac{\pi}{3} \leq 2x \leq \frac{2\pi}{3}$$

$$\Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{3} \Rightarrow x \in \left[-\frac{\pi}{6}, \frac{\pi}{3} \right]$$

24. (d) : Given $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is even.

$$\begin{aligned} \therefore f(x) &= f(-x) \Rightarrow \frac{a^x - 1}{x^n(a^x + 1)} \\ &= \frac{a^{-x} - 1}{(-x)^n(a^{-x} + 1)} = \frac{1 - a^x}{(-x)^n(1 + a^x)} = \frac{-(a^x - 1)}{(-1)^n x^n(a^x + 1)} \\ &\Rightarrow 1 = \frac{-1}{(-1)^n} \end{aligned}$$

Clearly the above relation is true for $n = 3$

25. (b) : Let $a, b \in A$ such that $(a, b) \in R_1 \cap R_2$.

$(a, b) \in R_1 \cap R_2 \Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$

$\Rightarrow aR_1b$ and aR_2b

$\Rightarrow bR_1a$ and bR_2a [$\because R_1$ and R_2 are symmetric]

$\Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2 \Rightarrow (b, a) \in R_1 \cap R_2$

Hence $R_1 \cap R_2$ is symmetric

26. (d) : Given, $A = \{1, 2, 3\}$ and $R = \{(a, b) : |a^2 - b^2| \leq 5\}$.

As $1^2 - 1^2 = 0 \Rightarrow |1^2 - 1^2| = 0 < 5 \therefore (1, 1) \in R$ i.e., $1R1$

$2^2 - 2^2 = 0 \Rightarrow |2^2 - 2^2| = 0 < 5 \therefore (2, 2) \in R$ i.e., $2R2$

$3^2 - 3^2 = 0 \Rightarrow |3^2 - 3^2| = 0 < 5 \therefore (3, 3) \in R$ i.e., $3R3$

$1^2 - 2^2 = -3 \Rightarrow |1^2 - 2^2| = 3 < 5 \therefore (1, 2) \in R$ i.e., $1R2$

$1^2 - 3^2 = -8 \Rightarrow |1^2 - 3^2| = 8 > 5 \therefore (1, 3) \notin R$ i.e., $1 \not R 3$

$2^2 - 3^2 = -5 \Rightarrow |2^2 - 3^2| = 5 \therefore (2, 3) \in R$ i.e., $2R3$

Similarly, $(2, 1) \in R$ i.e., $2R1$, $(3, 2) \in R$ i.e., $3R2$

Thus $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (2, 1), (3, 2)\}$

Now $R^{-1} = \{(y, x) : (x, y) \in R\}$

$= \{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 2), (2, 3)\} = R$

Domain of $R = \{x : (x, y) \in R\} = \{1, 2, 3\}$

Range of $R = \{y : (x, y) \in R\} = \{1, 2, 3\}$.

27. (c) : Let $y = f(x) = (x + 1)^2 - 1$

$$\Rightarrow x = \sqrt{y + 1} - 1$$

$$\Rightarrow f^{-1}(y) = \sqrt{y + 1} - 1 \text{ i.e., } f^{-1}(x) = \sqrt{x + 1} - 1$$

Now, $f(x) = f^{-1}(x)$

$$\Rightarrow (x + 1)^2 - \sqrt{x + 1} = 0 \Rightarrow \sqrt{x + 1} \left\{ (x + 1)^{3/2} - 1 \right\} = 0$$

$$\Rightarrow x + 1 = 0 \text{ or } (x + 1)^{3/2} = 1 \Rightarrow x = -1 \text{ or } x = 0$$

Thus $\{x : f^{-1}(x) = f(x)\} = \{-1, 0\}$

28. (b) : Let $x = y \cos \frac{2\pi}{3} = z \cos \frac{4\pi}{3} = \frac{1}{k}$ (say)

$$\therefore \frac{1}{x} = k, \frac{1}{y} = k \cos \frac{2\pi}{3} = -\frac{k}{2} \text{ and}$$

$$\frac{1}{z} = k \cos \frac{4\pi}{3} = k \cos \left(\pi + \frac{\pi}{3} \right) = -k \cos \frac{\pi}{3} = -\frac{k}{2}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = k - \frac{k}{2} - \frac{k}{2} = 0$$

$$\Rightarrow xy + yz + zx = 0$$

29. (b) : The given functions are

$$g(x) = 1 + x - [x] \text{ and } f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

$$\text{Now, } g(x) = 1 + x - [x] = 1 + (x - [x]) = 1 + \{x\} \geq 1 \quad [\because 0 \leq \{x\} < 1]$$

$$\Rightarrow g(x) > 0 \text{ for all } x$$

$$\therefore f(g(x)) = 1 \text{ for all } x$$

30. (c) : Here, $4 < x < 6 \Rightarrow 2 < \frac{x}{2} < 3 \therefore \left\lfloor \frac{x}{2} \right\rfloor = 2$

$$\text{So, } f(x) = x + \left\lfloor \frac{x}{2} \right\rfloor = x + 2, x \in (4, 6)$$

Let for $f(x) = x + \left\lfloor \frac{x}{2} \right\rfloor = x + 2, x \in (4, 6)$ there exists

$y \in (6, 8)$ such that $f(x) = y$

$$\Rightarrow y = x + 2 \Rightarrow x = y - 2.$$

$$\therefore f^{-1}(y) = y - 2 \Rightarrow f^{-1}(x) = x - 2.$$

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Challenging PROBLEMS



DM

Sequences and Series



1. The first term of a finite G.P. of real numbers is positive, and the sum of the series is negative, then a possible number of terms in the series is

- (a) 20 (b) 21 (c) 23 (d) 25

2. $\sum_{k=1}^{100} (-1)^k k^2 =$

- (a) 5000 (b) 5050 (c) 6000 (d) 6050

3. Simplify: $(0^3 - 350)(1^3 - 349)(2^3 - 348) \dots (349^3 - 1)(350^3 - 0)$

- (a) 2 (b) 1 (c) 0 (d) 5

4. The sum of the infinite series

$$1 + \frac{1}{2} + \frac{1}{10} + \frac{1}{20} + \frac{1}{100} + \dots \text{ is}$$

- (a) $\frac{5}{2}$ (b) $\frac{5}{3}$ (c) $\frac{5}{4}$ (d) 1

5. Let $p = \sum_{k=1}^{\infty} \frac{1}{k^2}$, $q = \sum_{k=1}^{\infty} \frac{1}{k^3}$, then $\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{(j+k)^3} =$

- (a) $p + q$ (b) $p - q$ (c) pq (d) p/q

6. Set A consists of m consecutive integers whose sum is $2m$ and set B consists of $2m$ consecutive integers whose sum is m . The difference between the greatest element of A and the greatest element of B is 1051, then $m =$

- (a) 2105 (b) 1025 (c) 5012 (d) 2501

7. Given that $x_1 = 211$, $x_2 = 375$, $x_3 = 420$, $x_4 = 523$ and $x_n = x_{n-1} - x_{n-2} + x_{n-3} - x_{n-4}$, $n \geq 5$, then

$$x_{531} + x_{753} + x_{975} =$$

- (a) 898 (b) 1010 (c) 1321 (d) 1430

8. Let the sequence of integers $a_1, a_2, \dots$ be defined by $a_1 = 2$, $a_2 = 3$, $a_3 = 7$, $a_4 = 43$ etc. In general $a_{n+1} = 1 + a_1 a_2$

$$\dots a_n, n \geq 1. \text{ If } S_n = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \text{ and}$$

$$P_n = \frac{1}{a_1 a_2 a_3 \dots a_n}, \text{ then } S_{21} + P_{21} =$$

- (a) 1 (b) 21 (c) 20 (d) $\frac{1}{20}$

9. Define two sequences of rational numbers as follows.

$$\text{Let } a_0 = 2, b_0 = 3 \text{ and } a_{n+1} = \frac{a_n^2}{b_n} \text{ and } b_{n+1} = \frac{b_n^2}{a_n}, n \geq 0.$$

$$\text{If } b_8 = \frac{3^p}{2^q}, \text{ then } p - q =$$

- (a) 0 (b) 1 (c) -1 (d) 2

10. The sequence $a_0, a_1, \dots$ satisfies $a_n = ka_{n-1} - a_{n-2}$, $n \geq 2$.

Then sequence $\langle a_n \rangle$ is periodic for $k = \sqrt{2}$ with period

- (a) 7 (b) 8 (c) 14 (d) 16

11. Sum $S = \sum_{n=1}^{\infty} \frac{(7n+32) \cdot 3^n}{n(n+2) \cdot 4^n}$ is

- (a) $\frac{31}{2}$ (b) $\frac{32}{2}$ (c) $\frac{33}{2}$ (d) $\frac{34}{2}$

12. Let $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ and

$$T_n = \frac{1}{(n+1)H_n H_{n+1}}, \text{ then } T_1 + T_2 + T_3 + \dots =$$

- (a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

13. Let a, b be fixed +ve integers. The sequence $\langle x_n \rangle$ is given by the recursive relation

$$x_{n+1} = x_n + a + \sqrt{b^2 + 4ax_n}, n \geq 0 \text{ and } x_0 = 0.$$

$$\text{Then } \lim_{n \rightarrow \infty} \frac{x_n}{n^2} =$$

- (a) a (b) b (c) $a + b$ (d) ab

14. Sum $S = \sum_{k=1}^{\infty} \frac{k^3 + k}{2^k}$ is

- (a) 21 (b) 24 (c) 26 (d) 28



15. The sequence $\langle a_n \rangle$ satisfies $a_1 = 1$ and

$$5^{a_{n+1}-a_n} = \frac{3}{3n+2} + 1 \text{ for } n \geq 1. \text{ The least integer}$$

$m (> 1)$ so that a_m is an integer is

- (a) 25 (b) 31 (c) 41 (d) 52

16. For any positive integer n , let $\bar{n}$ denote the closest

integer to $\sqrt{n}$, then $\sum_{n=1}^{\infty} \frac{2^{\bar{n}} + \left(\frac{1}{2}\right)^{\bar{n}}}{2^n} =$

- (a) 2 (b) 3 (c) $1/2$ (d) $1/3$

17. The value of $\sum_{k=1}^{\infty} \frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$ is
(a) 2 (b) $1/2$ (c) 3 (d) $1/3$

18. Let $a_0, a_1, a_2, \dots, a_n$ be a sequence of numbers satisfying $(3 - a_{n+1})(6 + a_n) = 18$ and $a_0 = 3$.

If $\frac{1}{a_0} + \frac{1}{a_1} + \dots + \frac{1}{a_n} = \frac{2^{n+2} - n - 3}{\lambda}$, then $\lambda =$

- (a) 1 (b) 2 (c) 3 (d) 4

19. If $\sum_{r=1}^{20} \left(\frac{r^2 - \frac{1}{2}}{r^4 + \frac{1}{4}} \right) = \frac{\lambda}{\mu}$, then $\mu - \lambda - 20 =$

(λ, μ being in lowest terms)

- (a) 20 (b) 21 (c) 22 (d) 23

20. $a, b, c \in \mathbb{R}^+$, the least value of $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a}$ is

- (a) $11/4$ (b) $21/4$ (c) $30/4$ (d) $5/4$

SOLUTIONS

1. (a): $S = \frac{a(r^n - 1)}{r - 1}$. If $r \geq 0$, then every term is non-negative and hence S is non-negative i.e., a contradiction to given data. Hence, r must be negative $\Rightarrow r^n - 1$ must be +ve i.e., n should be even.

2. (b):

$$\sum_{k=1}^{100} (-1)^k \cdot k^2 = (2^2 - 1^2) + (4^2 - 3^2) + \dots + (100^2 - 99^2) \\ = (2 + 1) + (4 + 3) + \dots + (100 + 99) \\ = 1 + 2 + 3 + 4 + \dots + 100 = 5050.$$

3. (c): Notice that 8th term is 0.

4. (b): Given, $\text{sum} = \left(1 + \frac{1}{10} + \frac{1}{100} + \dots\right) \\ + \left(\frac{1}{1} + \frac{1}{20} + \frac{1}{200} + \dots\right) = \frac{5}{3}.$

5. (b): Required sum $= \sum_{j=1}^{\infty} \left(\frac{1}{(j+1)^3} + \frac{1}{(j+2)^3} + \dots \right)$

$$= \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \dots$$

$$+ \frac{1}{3^3} + \frac{1}{4^3} + \dots$$

$$+ \frac{1}{4^3} + \frac{1}{5^3} + \dots$$

$$= \sum_{n=2}^{\infty} \frac{n-1}{n^3} = \sum_{n=2}^{\infty} \frac{1}{n^2} + 1 - \sum_{n=2}^{\infty} \frac{1}{n^3} = 1 - p - q$$

6. (a): Set $A = \{a, a+1, \dots, a+m-1\}$

$$\therefore \text{Sum}, 2m = \frac{m}{2}[2a + (m-1)] \text{ i.e., } a = \frac{5-m}{2}$$

$$\text{Also, set } B = \{b, b+1, \dots, b+2m-1\}$$

$$\text{Now sum}, m = \frac{2m}{2}[2b + (2m-1)] \text{ i.e., } b = 1 - m$$

$$A_{\max} = a + m - 1 = \frac{m+3}{2}, B_{\max} = b + 2m - 1 = m$$

$$\text{According to the question, } \left| \frac{m+3}{2} - m \right| = 1051$$

$$\Rightarrow m = 2105.$$

7. (a): $x_5 = 267$. Iterating n to $(n-1)$ and simplifying, we have $x_n = -x_{n-5}$ i.e., $x_n = (-1)^k x_{n-5k}$
Hence, $x_{531} = (-1)^{106} x_1, x_{753} = (-1)^{150} x_3, x_{975} = (-1)^{194} x_5$

8. (a): Let $t_n = S_n + P_n$, then $t_1 = 1, t_2 = 1, t_3 = 1, \dots$. So, a general guess is $t_n = 1$.

$$\text{Indeed } t_n = S_n + P_n = S_{n+1} + P_{n+1} = t_{n+1}$$

$$\text{Hence, } t_{n+1} = t_n = t_{n-1} = \dots = t_1 = 1$$

$$\text{i.e., } S_{21} + P_{21} = t_{21} = 1$$

9. (b): $a_{n+1} b_{n+1} = a_n b_n = a_{n-1} b_{n-1} = \dots = a_0 b_0 = 6$

$$\text{Hence, } b_{n+1} = \frac{b_n^3}{6}$$

$$\text{So, } b_8 = \frac{b_0^8}{6^{3^7} \cdot 6^{3^6} \cdot \dots \cdot 6^{3^2} \cdot 6^{3^1} \cdot 6} = \frac{3^{6561}}{6^{3280}} = \frac{3^{3281}}{2^{3280}}$$

10. (b): With $k = \sqrt{2}$, $a_2 = \sqrt{2}a_1 - a_0, a_3 = a_1 - \sqrt{2}a_0$

$$\text{Similarly, } a_8 = a_0, a_9 = a_1$$

$$\therefore \text{Period} = 8$$

11. (c): Using partial fractions, given sum S is

$$S = \sum_{n=1}^{\infty} \left(\frac{4^2}{n} - \frac{3^2}{n+2} \right) \cdot \frac{3^n}{4^n}$$

$$\text{Now, notice that } \frac{-3^2}{k+2} - \frac{3^k}{4^k} = \frac{-4^2}{k+2} - \frac{3^{k+2}}{4^{k+2}}$$





Hence, terms in S , cancels after two-two terms each.

$$\therefore S = \frac{1}{1} \cdot 3 \cdot 4 + \frac{1}{2} \cdot 3 \cdot 3 = \frac{33}{2}$$

12. (a) : Notice that $H_{n+1} = H_n + \frac{1}{n+1}$

$$\text{Hence, } T_n = \frac{1}{H_n} - \frac{1}{H_{n+1}}$$

$$\text{So, } T_1 + T_2 + T_3 + \dots = \frac{1}{H_1} = 1$$

13. (a) : The given relation can be re-written as

$$\sqrt{b^2 + 4ax_{n+1}} = 2a + \sqrt{b^2 + 4ax_n}$$

i.e., $y_{n+1} = 2a + y_n$, where $y_n = \sqrt{b^2 + 4ax_n}$

i.e., y_n is an A.P. series with common diff. = $2a$

Hence, $x_n = an^2 + bn$.

14. (d) : Given $S = \sum_{k=1}^{\infty} \frac{k^3 + k}{2^k} = 2$

$$S = 2 + \sum_{k=1}^{\infty} \frac{3k^2 + 3k + 2}{2^k} = 2 + 3 \sum_{k=1}^{\infty} \frac{k^2}{2^k} + 3 \sum_{k=1}^{\infty} \frac{k}{2^k} + 2 \sum_{k=1}^{\infty} \frac{1}{2^k}$$

$$\text{Now, } \sum_{k=1}^{\infty} kx^k = \frac{x}{(1-x)^2} \text{ and } \sum_{k=1}^{\infty} k^2 x^k = \frac{x(x+1)}{(1-x)^3}$$

Putting $x = 1/2$ in these results

$$\sum_{k=1}^{\infty} \frac{k^2}{2^k} = 2, \sum_{k=1}^{\infty} \frac{k}{2^k} = 6 \text{ and } \sum_{k=1}^{\infty} \frac{1}{2^k} = 1$$

Hence, $S = 2 + 3(6) + 3(2) + 2(1) = 28$.

15. (c) : $5^{a_{n+1}-a_n} = \frac{3n+5}{3n+2}, 5^{a_n-a_{n-1}} = \frac{3n+2}{3n-1}, \dots$

Multiplying all, $5^{a_n-a_1} = \frac{3n+5}{5} \Rightarrow a_n = \log_5(3n+2)$

So, $a_m = \log_5(3m+2)$ is integer for least $m = 41$.

16. (b) : Notice that $\left(r - \frac{1}{2}\right)^2 = r^2 - r + \frac{1}{4}$

$$\text{and } \left(r + \frac{1}{2}\right)^2 = r^2 + r + \frac{1}{4}$$

So $\bar{r} = r$ iff $r^2 - r + 1 \leq n \leq r^2 + r + 1$

Hence, required sum is

$$\sum_{r=1}^{\infty} \sum_{\bar{r}=r}^{\left(r+\frac{1}{2}\right)^2} \frac{1}{2^{\bar{r}}} = \sum_{r=1}^{\infty} \sum_{n=r^2-r+1}^{r^2+r+1} \frac{2^r + 2^{-r}}{2^n} = 3$$

17. (a) : Given expression can be expressed as

$$\sum_{k=1}^{\infty} \left(\frac{3^k}{3^k - 2^k} - \frac{3^{k+1}}{3^{k+1} - 2^{k+1}} \right)$$

18. (c) : As reqd. sum is of form $\frac{1}{a_n}$, lets replace $\frac{1}{a_n}$

with b_n to get the new recurrence relation,

$$3b_{n+1} - 6b_n = 1 \Rightarrow b_{n+1} + \frac{1}{3} = 2 \left(b_n + \frac{1}{3} \right)$$

Let $b_n + \frac{1}{3} = c_n$, then $c_{n+1} = 2c_n$
(G.P. with common ratio 2)

$$\text{Hence, } b_n = \frac{1}{3} (2^{n+1} - 1)$$

and so, $\sum_{r=0}^n \frac{1}{a_r} = \frac{1}{3} (2^{n+2} - n - 3)$ i.e., $\lambda = 3$

$$\text{19. (b) : } T_r = \frac{r^2 - \frac{1}{2}}{r^4 + \frac{1}{4}} = \frac{r - \frac{1}{2}}{r^2 - r + \frac{1}{4}} - \frac{(r+1) - \frac{1}{2}}{(r+1)^2 - (r+1) + \frac{1}{4}}$$

$$\text{Hence, } \sum_{r=1}^{20} T_r = \sum_{r=1}^{20} (S_r - S_{r+1}) = S_1 - S_{21} = 1 - \frac{41}{841} = \frac{800}{841}$$

$$\left(\text{where } S_r = \frac{r - \frac{1}{2}}{r^2 - r + \frac{1}{4}} \right)$$

20. (d) : Apply A.M. $\geq$ G.M. on the quantities

$$\frac{a^3}{4b}, \frac{b}{8c^2}, \frac{1}{2a}, \frac{c}{4a}, \frac{c}{4a}$$



PUZZLE CORNER

ANSWER · MAY 2019



$\frac{3}{2}$	5	$\frac{5}{1}$	$\frac{216}{3}$	6	4
$\frac{72}{1}$	$\frac{13}{6}$	5	$\frac{2}{4}$	2	3
6	3	4	$\frac{2}{2}$	1	$\frac{1}{5}$
3	4	$\frac{2}{2}$	1	$\frac{60}{5}$	6
$\frac{8}{4}$	2	$\frac{3}{6}$	$\frac{11}{5}$	3	$\frac{2}{1}$
$\frac{5}{5}$	1	3	6	4	2

MATH archives



Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

- The constant term in the expansion of $\left(x + x^2 + \frac{1}{x} + \frac{1}{x^2}\right)^{15}$ is
(a) $(^{15}C_0)^2$ (b) $^{30}C_{15}$
(c) $2 \cdot ^{15}C_0$ (d) $\sum_{r=0}^5 (^{15}C_{5+r}) (^{15}C_{3r})$
- Let S be the set of 6-digit numbers $a_1 a_2 a_3 a_4 a_5 a_6$ (all digits distinct) where $a_1 > a_2 > a_3 > a_4 < a_5 < a_6$. Then $n(S)$ is equal to
(a) 210 (b) 2100 (c) 4200 (d) 420
- The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α . If R is the circumradius of the triangle ABC , then the height of the tower is
(a) $R \sin \alpha$ (b) $R \cos \alpha$
(c) $R \cot \alpha$ (d) $R \tan \alpha$
- The number of ordered pairs (x, y) satisfying the system of equations given by $\sin x + \sin y = \sin(x + y)$ and $|x| + |y| = 1$ is
(a) 2 (b) 4
(c) 6 (d) None of these
- The number of real solutions of the equation $\log_2(x+2) = 6 - x$ is
(a) 0 (b) 1 (c) 2 (d) 3
- The set of values of x for which $(x^2 + x + 1)^x < 1$ is
(a) $(-\infty, -1)$ (b) $(-\infty, 1)$
(c) $(-\infty, 0)$ (d) None of these
- If $f(x) = \int_0^x (e^t + 3) dt$, then which of the following is always true $\forall x \in \mathbb{R}$?
(a) $f(x-3) - f(x-1) \leq 11$
(b) $f(x+1) - f(x-1) \leq 11$
(c) $f(x+3) - f(x-5) \geq 24$
(d) $f(x) + f(x+1) < -21$
- The equation of curve passing through $(1, 1)$ in which the subtangent is always bisected at the origin is
(a) $y^2 = x$ (b) $2x^2 - y = 1$
(c) $x^2 + y^2 = 2$ (d) $x + y = 2$
- Three friends whose ages form a G.P. divide a certain sum of money in proportion to their ages. If they do that three years later, when the youngest is half the age of the oldest, then he will receive ₹ 105 more than he gets now, and the middle friend will get ₹ 15 more than now, then the age of the youngest friend is
(a) 27 (b) 18 (c) 14 (d) 12
- The first two terms of an H.P. are $\frac{2}{5}$ and $\frac{12}{23}$. The value of the largest term of the H.P. is
(a) $\frac{72}{73}$ (b) 6
(c) $\frac{1}{6}$ (d) None of these



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SOLUTIONS

$$1. (d): \left(x + x^2 + \frac{1}{x} + \frac{1}{x^2}\right)^{15} = (x + x^2)^{15} \left(1 + \frac{1}{x^3}\right)^{15}$$

$$= x^{15} \{ {}^{15}C_0 + {}^{15}C_1 x + \dots + {}^{15}C_{15} x^{15} \}$$

$$\left\{ {}^{15}C_0 + {}^{15}C_1 \frac{1}{x^3} + {}^{15}C_2 \frac{1}{x^6} + \dots + {}^{15}C_{15} \frac{1}{x^{45}} \right\}$$

Constant terms add up to

$${}^{15}C_5 {}^{15}C_0 + {}^{15}C_6 {}^{15}C_3 + {}^{15}C_7 {}^{15}C_6 + {}^{15}C_8 {}^{15}C_9$$

$$+ {}^{15}C_9 {}^{15}C_{12} + {}^{15}C_{10} {}^{15}C_{15}$$

2. (b): First, 6 distinct digits can be selected in ${}^{10}C_6$ ways. Now the positive of smallest digit in them is fixed i.e. position 4. Of the remaining 5 digits, two digits can be selected in ${}^5C_2$ ways. These two digits can be placed to the right of 4th position in one way only. The remaining three digits to the left of 4th position are in the required order automatically

So $n(S) = {}^{10}C_6 \times {}^5C_2 = 210 \times 10 = 2100$.

3. (d)

4. (c): Given, $\sin x + \sin y = \sin(x + y)$

$$\Rightarrow 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right) = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x+y}{2}\right)$$

$$\Rightarrow \text{Either } \sin\left(\frac{x+y}{2}\right) = 0 \text{ or } \cos\left(\frac{x-y}{2}\right) = \cos\left(\frac{x+y}{2}\right)$$

$$\Rightarrow x = 0 \text{ or } y = 0 \text{ or } x + y = 0$$

Other values are not possible as $|x| + |y| = 1$

Hence, the possible solutions are $(0, \pm 1)$, $(\pm 1, 0)$,

$\left(\pm \frac{1}{2}, \mp \frac{1}{2}\right)$ i.e., six solutions.

5. (b): The curve $y = \log_7(x+2)$ and $y = 6 - x$ intersect at only one point.

6. (a)

7. (c): $f'(x) = e^x + 3$

$$\therefore f'(x) \geq 3$$

$$\text{Also, } \frac{f(x+3) - f(x-5)}{8} = f'(c) \geq 3$$

$$\Rightarrow f(x+3) - f(x-5) \geq 24$$

8. (a): TM is subtangent where $T \equiv \left(x - y \frac{dx}{dy}, 0\right)$ and $M \equiv (x, 0)$

$$\text{So, } \frac{1}{2} \left(x - y \frac{dx}{dy} + x\right) = 0$$

$$\Rightarrow 2x - y \frac{dx}{dy} = 0 \quad \text{or} \quad 2 \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$2 \ln y = \ln cx \Rightarrow y^2 = cx.$$

Since the curve passes through $(1, 1)$.

$$\therefore c = 1 \Rightarrow y^2 = x$$

9. (b): Let the ages of friends be $a, ar, ar^2, r > 1$ and money distributed is x, xr, xr^2 respectively.

$$\Rightarrow (1 + r + r^2)x = (a(1 + r + r^2) + 9)t$$

$$\text{Also after 3 years } ar^2 + 3 = 2(a + 3)$$

$$\Rightarrow a(r^2 - 2) = 3$$

$$\text{Now, } x + 105 = (a + 3)t \quad \text{and}$$

$$xr + 15 = (ar + 3)t$$

(Given)

$$\Rightarrow (1 + r)x + 120 = (a(1 + r) + 6)t$$

$$\Rightarrow 2(a + 3)t = r^2x - 120 = 2x + 210$$

$$\Rightarrow (r^2 - 2)x = 330$$

$$\text{Using } (r^2 - 2)x = 330, a(r^2 - 2) = 3 \Rightarrow x = 110a$$

$$\Rightarrow 110a + 105 = (a + 3)t \Rightarrow 5(22a + 21) = (a + 3)t$$

$$\text{and } 110ar + 15 = (ar + 3)t$$

$$\Rightarrow 5(22ar + 3) = (ar + 3)t$$

$$\Rightarrow \frac{22a + 21}{22ar + 3} = \frac{a + 3}{ar + 3} \Rightarrow 5ar = 7a + 6$$

Where $5ar = 7a + 2(a(r^2 - 2))$ [Using $a(r^2 - 2) = 3$]

$$\Rightarrow 2r^2 - 5r + 3 = 0 \Rightarrow r = \frac{3}{2}. \quad \text{Hence } a = 12 \text{ years.}$$

10. (b): $\frac{5}{2}$ and $\frac{23}{12}$ are in A.P., $d = \frac{-7}{12}$

$$T_n = \frac{37 - 7n}{12} > 0 \Rightarrow n \leq 5$$

$\Rightarrow$ 5th term has least positive value.

Therefore the value of the largest term of H.P. = 6.



COMIC CAPSULE

Mathematical Tau Bhatija





SOLVED PAPER 2019

Kerala PET

1. The axis of the parabola $x^2 + 6x + 4y + 5 = 0$ is
(a) $x = 0$ (b) $y = 1$
(c) $x + 3 = 0$ (d) $y = 4$
(e) $y + 2 = 0$
2. The distance between the foci of the ellipse $\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$ is
(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $3\sqrt{5}$ (d) $9\sqrt{5}$ (e) $7\sqrt{5}$
3. The value of k , if the circles $2x^2 + 2y^2 - 4x + 6y = 3$ and $x^2 + y^2 + kx + y = 0$ cut orthogonally is
(a) 2 (b) 3 (c) 4 (d) 5 (e) 1
4. The circle passing through $(1, -2)$ and touching the x -axis at $(3, 0)$ also passes through the point
(a) $(2, -5)$ (b) $(-5, -2)$
(c) $(-2, 5)$ (d) $(-5, 2)$
(e) $(5, -2)$
5. If α and β are the roots of the equation $x^2 + \alpha x + \beta = 0$, then
(a) $\alpha = -1, \beta = -2$ (b) $\alpha = 0, \beta = 1$
(c) $\alpha = -2, \beta = 0$ (d) $\alpha = -2, \beta = 1$
(e) $\alpha = 1, \beta = -2$
6. If $\vec{a} = (1, 1, -1)$, $\vec{b} = (-1, 2, 1)$ and $\vec{c} = (-1, 2, -1)$, then $|(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})|$ is
(a) 2 (b) 4 (c) 6 (d) 8 (e) 10
7. A particle is displaced from the point $(2, 1, -1)$ to the point $(4, 3, -4)$ by the force $2\hat{i} + 4\hat{j} - 5\hat{k}$. Then the work done by the force is
(a) 16 (b) 27 (c) 36 (d) 48 (e) 52
8. The value of m if the vectors $4\hat{i} - 3\hat{j} + 5\hat{k}$ and $m\hat{i} - 4\hat{j} + \hat{k}$ are perpendicular, is
(a) $-\frac{15}{4}$ (b) $-\frac{17}{4}$ (c) $-\frac{19}{4}$ (d) 0 (e) $\frac{11}{4}$
9. If A and B are two matrices such that $3A + B = \begin{pmatrix} 9 & 11 & 3 \\ 12 & 14 & 19 \end{pmatrix}$ and $2A - 3B = \begin{pmatrix} -16 & 11 & 2 \\ -3 & -22 & 9 \end{pmatrix}$. Then the matrix B is
(a) $\begin{pmatrix} 6 & -1 & 0 \\ 3 & 8 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$
(c) $\begin{pmatrix} 8 & 0 & -1 \\ 3 & 1 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 5 & 3 & -1 \\ 0 & 1 & 2 \end{pmatrix}$
(e) $\begin{pmatrix} 1 & -3 & 4 \\ 3 & 0 & 2 \end{pmatrix}$
10. If a, b and c are distinct reals and the determinant $\begin{vmatrix} a^3+1 & a^2 & a \\ b^3+1 & b^2 & b \\ c^3+1 & c^2 & c \end{vmatrix} = 0$, then the product abc is
(a) -1 (b) 0 (c) 1 (d) 2 (e) 3
11. If (x, y, z) is the solution of the equations $x - y - 2z = 3$, $2x + y + 4z = 5$, $4x - y - 2z = 11$ then the value of y equals
(a) 0 (b) $-\frac{1}{2}$ (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$ (e) -1
12. If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is the inverse of the matrix $\begin{pmatrix} 1 & 5 \\ 7 & -3 \end{pmatrix}$, then d equals
(a) $-\frac{1}{38}$ (b) $-\frac{7}{38}$ (c) $\frac{3}{38}$ (d) $\frac{5}{38}$ (e) $\frac{9}{38}$
13. If $f: R \rightarrow R$ is a function defined by $f(x) = \sin x$, then which of the following is true?
(a) f is 1-1 but not onto
(b) f is onto but not 1-1

- (c) f is both 1-1 and onto
 (d) f is neither 1-1 nor onto
 (e) f has finite number of zeros

14. Consider the set $M = \{1, 2, 3\}$ along with the relation $R = \{(1, 2), (1, 1), (3, 1), (3, 4), (3, 3), (4, 3)\}$. Which of the following statements is true?
 (a) The relation is symmetric but not transitive
 (b) The relation is transitive but not symmetric
 (c) The relation is both symmetric and transitive
 (d) The relation is neither symmetric nor transitive
 (e) The relation is reflexive

15. Let $z_1 = 1 + i\sqrt{3}$ and $z_2 = 1 + i$, then $\arg\left(\frac{z_1}{z_2}\right)$ is
 (a) $\frac{5\pi}{12}$ (b) $\frac{7\pi}{12}$ (c) $\frac{11\pi}{12}$ (d) $\frac{3\pi}{12}$
 (e) Not defined

16. The complex number $\sqrt{2}\left[\sin\frac{\pi}{8} + i\cos\frac{\pi}{8}\right]^6$ represents
 (a) $-i$ (b) i (c) $1 - i$
 (d) $1 + i$ (e) $1 + 2i$

17. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of
 $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$ is
 (a) 18 (b) 54 (c) 6 (d) 19 (e) 12

18. The value of $\tan\left[\sin^{-1}\frac{-1}{\sqrt{2}}\right]$ is
 (a) -1 (b) 0 (c) 1 (d) Infinity
 (e) 2

19. If $\sin^{-1}x + \cos^{-1}2x = \frac{\pi}{6}$, then the value of x is
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1 (e) $\sqrt{2}$

20. If $x = 2\cos t - \cos 2t$ and $y = 2\sin t - \sin 2t$, then $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$ is
 (a) -1 (b) 0 (c) $\frac{1}{2}$ (d) 1 (e) 3

21. The equation of the tangent to the curve given by $x^2 + 2x - 3y + 3 = 0$ at the point $(1, 2)$ is
 (a) $4x - 3y - 2 = 0$ (b) $3y - 4x - 2 = 0$
 (c) $4x + 3y + 2 = 0$ (d) $4x + 3y - 2 = 0$
 (e) $4y - 3x + 2 = 0$

22. The value of $\lim_{x \rightarrow \infty} \frac{x^3 \sin\left(\frac{1}{x}\right) - 2x^2}{1 + 3x^2}$ is
 (a) 0 (b) $\frac{1}{3}$ (c) -1 (d) $-\frac{2}{3}$ (e) $-\frac{1}{3}$

23. The maximum value of $y = \left(\frac{1}{x}\right)^x$, $x > 0$ is
 (a) $e^{1/e}$ (b) e^e (c) 1 (d) Infinity
 (e) 0

24. The value of the integral $\int_0^{\pi} \frac{\cos x}{1 + \sin^2 x} dx$ is
 (a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) π (e) 2π

25. The area enclosed between the curves $y = 2x^2 + 1$ and $y = x^2 + 5$ is
 (a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{16}{3}$ (d) $\frac{32}{3}$ (e) $\frac{1}{3}$

26. The solution of the differential equation $5ydx = 2x dy$ passing through the point $(1, 1)$ is
 (a) $2 \ln x = 5 \ln y$ (b) $5 \ln x = 2 \ln y$
 (c) $\ln(y + x) = 2$ (d) $\ln(1 + xy) = 0$
 (e) $3 \ln x = 5 \ln y$

27. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and $y = 0$ is
 (a) 4 (b) 12 (c) 3 (d) 14 (e) 1

28. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is
 (a) 22.0 (b) 20.5 (c) 25.5 (d) 23.2 (e) 24.0

29. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
 (a) $\frac{15}{2}$ (b) 6 (c) $\frac{13}{2}$ (d) $\frac{5}{2}$ (e) $\frac{11}{2}$

30. If the mean of the first n odd numbers is $\frac{n^2}{81}$, then n equals
 (a) 9 (b) 18 (c) 27 (d) 81 (e) 52

31. A bag contains 5 red balls and some blue balls. If the probability of drawing blue ball is double that of red ball, the number of blue balls must be
 (a) 10 (b) 15 (c) 20 (d) 25 (e) 30

32. A pair of fair dice are rolled together. The probability of getting a total of 8 is
 (a) $\frac{1}{9}$ (b) $\frac{5}{36}$ (c) $\frac{7}{36}$ (d) $\frac{11}{36}$ (e) $\frac{1}{36}$



33. In a chess tournament, assume that your probability of winning a game is 0.3 against level 1 players, 0.4 against level 2 players and 0.5 against level 3 players. It is further assumed that among the players 50% are at level 1, 25% are at level 2 and the remaining are at level 3. Suppose that you win the game. Then the probability that you had played with level 1 player is
(a) 0.3 (b) 0.4 (c) 0.5 (d) 0.6 (e) 0.2
34. A sum of ₹ 280 is to be used award four prizes. If each prize after the first prize is ₹ 20 less than its preceding prize, then the value of the fourth prize is
(a) 20 (b) 40 (c) 60 (d) 80 (e) 10
35. The coefficient of x^3 in the expansion of $(1 + x + 2x^2)(1 - 2x)^5$ is
(a) -20 (b) -40 (c) -60 (d) -80 (e) -100
36. The constant term in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$ is
(a) 60 (b) 180 (c) 240 (d) 360 (e) 420
37. If the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$; $2x + 3y + 4z = 5$ and through the point $(1, 2, 3)$ is $3(x^2 + y^2 + z^2) - 2x - 3y - 4z = C$, then the value of C is
(a) 11 (b) 22 (c) 36 (d) 41 (e) 54
38. The equation of the plane containing the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ is $a(x - \alpha) + b(y - \beta) + c(z - \gamma) = 0$, where $al + bm + cn$ is equal to
(a) 1 (b) -1 (c) 2 (d) 8 (e) 0
39. Let $f(x)$ and $g(x)$ be two differentiable functions for $0 \leq x \leq 1$ such that $f(0) = 2$, $g(0) = 0$, $f(1) = 6$. If there exists a real number c in $(0, 1)$ such that $f'(c) = 2g'(c)$, then $g(1)$ is equal to
(a) 0 (b) -1 (c) 4 (d) -2 (e) 2
40. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$ that is parallel to the x -axis is
(a) $y = 1$ (b) $y = 2$ (c) $y = 8$ (d) $y = 0$ (e) $y = 3$
41. The number 81 is the coefficient of x^k in the binomial expansion of $\left(x^2 + \frac{3}{x}\right)^4$, $x \neq 0$. Then the value of k equals
(a) -2 (b) 2 (c) -4 (d) 4 (e) 5
42. The possible number of arrangements starting with K of the word KALINGA is
(a) 300 (b) 330 (c) 360 (d) 390 (e) 370
43. A bag contains 3 black and 2 white balls. A ball is drawn at random and is put back in the bag along with one ball of the same colour. A ball is again drawn at random. What is the probability that it is white?
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$ (e) $\frac{2}{13}$
44. If A and B are two events associated with an experiment such that $P(A \cup B) = P(A \cap B)$, and $P(A) = 1/3$, then $P(B)$ equals
(a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{5}$
45. Three identical fair dice are rolled. The probability that the same number appears on each of them is
(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{36}$ (d) $\frac{1}{216}$ (e) $\frac{1}{9}$
46. Let $\omega \neq 1$ be a cube root of unity and $(1 + \omega)^7 = a + \omega$. Then the value of a is
(a) ω^2 (b) ω (c) $\frac{1}{2}$ (d) 1 (e) 0
47. Let $w = \frac{1-iz}{z-i}$. If $|w| = 1$, which of the following must be true?
(a) z lies inside the unit circle
(b) z lies on real axis
(c) z lies on imaginary axis
(d) z lies outside the unit circle
(e) $\text{Re } z < 0$
48. For $|z| \geq 2$, if $\left|z + \frac{1}{z}\right| \geq k$, the minimum possible value of k is
(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$ (e) 3
49. Let $\cot \theta = -\frac{5}{12}$ where $\frac{\pi}{2} < \theta < \pi$. Then the value of $\sin \theta$ is
(a) $-\frac{12}{13}$ (b) $-\frac{5}{13}$ (c) $\frac{12}{13}$ (d) $\frac{5}{13}$ (e) $\frac{7}{13}$
50. The value of $\tan \frac{\pi}{8}$ is
(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $1-\sqrt{2}$ (e) $-1-\sqrt{2}$
51. In an A.P., if 5th term is $\frac{1}{7}$ and 7th term is $\frac{1}{5}$, then the sum of first 35 terms is
(a) 9 (b) 18 (c) 36 (d) 72 (e) 83



52. In a G.P., $1, \frac{1}{2}, \frac{1}{4}, \dots$, when the first n number of terms are added, the sum is $\frac{1023}{512}$. Then the value of n is
(a) 10 (b) 12 (c) 14 (d) 16 (e) 18
53. If A.M. and G.M. of the roots of a quadratic equation are 8 and 5 respectively, then the quadratic equation is
(a) $x^2 + 8x + 5 = 0$ (b) $x^2 - 16x + 10 = 0$
(c) $x^2 - 16x + 25 = 0$ (d) $x^2 + 8x + 25 = 0$
(e) $x^2 + 10x + 15 = 0$
54. Given that the equation $x^2 - (2a + b)x + (2a^2 + b^2 - b + \frac{1}{2}) = 0$ has two real roots. The value of b is
(a) 1 (b) 2 (c) -1 (d) -2 (e) 0
55. If ${}^5P_r = {}^6P_{r-1}$, then the value of r is
(a) $r = 1$ (b) $r = 5$ (c) $r = 3$ (d) $r = 2$ (e) $r = 4$
56. If ${}^{nC}_{2017} = {}^{nC}_{2016}$, then ${}^{nC}_{4033}$ equals
(a) 1 (b) 2016 (c) 2017 (d) 2033 (e) 2019
57. The image of the point $P(2, 1)$ on the straight line $2x - 3y + 1 = 0$ is
(a) $(\frac{1}{3}, \frac{25}{13})$ (b) $(\frac{15}{13}, \frac{25}{13})$
(c) $(\frac{18}{13}, \frac{25}{13})$ (d) $(\frac{21}{13}, \frac{25}{13})$
(e) $(\frac{11}{13}, \frac{15}{13})$
58. If the centre of the circle inscribed in a square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is (a, b) , then $a + b$ is
(a) 11 (b) 9 (c) 7 (d) 5 (e) 4
59. The equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$ is
(a) $x = -1$ (b) $x = 1$
(c) $x = \frac{3}{2}$ (d) $x = -\frac{3}{2}$
(e) $x = 2$
60. The foci of the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$ are
(a) $(\pm 1, 0)$ (b) $(\pm \alpha, 0)$
(c) $(0, \pm 1)$ (d) $(0, \pm \alpha)$
(e) $(1, \pm \alpha)$
61. The domain of definition of the function $f(x) = \frac{\log_3(x+7)}{x^2 - 5x + 6}$ is
(a) $(-7, \infty) \setminus \{3, 2\}$ (b) $(-3, \infty) \setminus \{3, 2\}$
(c) $(-7, \infty) \setminus \{3\}$ (d) $(-3, \infty) \setminus \{3\}$
(e) $(-5, \infty) \setminus \{3\}$
62. Let $f(x) = 3x - 5$. The inverse of f is given by
(a) $\frac{1}{3x-5}$ (b) $\frac{x+5}{3}$
(c) $\frac{x-1}{3-\frac{1}{5}}$ (d) $\frac{x}{3} + \frac{1}{5}$
(e) $\frac{3}{x-5}$
63. Let $R = \{(a, b) : a \leq b^2\}$ be a relation on the set of all real numbers. Then R is
(a) symmetric but not transitive
(b) transitive but not symmetric
(c) both symmetric and transitive
(d) neither symmetric nor transitive
(e) having finite range
64. A unit vector $\hat{b}$ is coplanar with $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ and is perpendicular to $\hat{i} + \hat{j} + \hat{k}$. Then $\hat{b} \cdot \hat{i}$ equals
(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) 2 (e) 4
65. Suppose $\alpha\hat{a} + \alpha'\hat{j} + \gamma\hat{k}, \hat{i} + \hat{k}$ and $\gamma\hat{i} + \gamma'\hat{j} + \beta\hat{k}$ are coplanar where α, β and γ are positive constants. Then the product $\alpha\beta$ is
(a) γ (b) γ^2 (c) 2γ (d) $2\gamma^2$ (e) 3γ
66. The area of the triangle whose vertices are $A(1, -1, 2), B(2, 1, -1)$ and $C(3, -1, 2)$ is
(a) $\sqrt{7}$ (b) $\sqrt{11}$ (c) $\sqrt{13}$ (d) $\sqrt{15}$ (e) $\sqrt{10}$
67. Let $f(x) + 2f(\frac{1}{x}) = \frac{1}{x} - 5$. Then $\int_1^2 3f(x)dx$ equals
(a) $2 + \ln 2$ (b) $2 - \ln 2$
(c) 2 (d) $3 \ln 2$
(e) $\ln 2$
68. The value of $\lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{6n} \right]$ is
(a) $\ln 3$ (b) $\ln 6$ (c) e^3 (d) e^6 (e) $\ln 2$
69. Let $f(x)$ be differentiable and $\int_0^{t^2} xf(x)dx = \frac{1}{2}t^4$ for all t . Then the value of $f(17)$ is
(a) 17 (b) 1 (c) $1/17$ (d) $17/2$ (e) 19





82. One of the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is
 (a) (3, 0) (b) (4, 0)
 (c) (5, 0) (d) (9, 0)
 (e) (2, 0)
83. If the circles $x^2 + y^2 - 8x - 6y + c = 0$ and $x^2 + y^2 - 2y + d = 0$ cut orthogonally, then $c + d$ equals
 (a) 6 (b) 4 (c) 2 (d) 0 (e) 1
84. The points with position vector $60\hat{i} + 3\hat{j}$, $40\hat{i} - 8\hat{j}$ and $a\hat{i} - 52\hat{j}$ are collinear if
 (a) $a = -10$ (b) $a = 40$
 (c) $a = 20$ (d) $a = 10$
 (e) $a = -40$
85. The area enclosed within the curve $|x| + |y| = 1$ is
 (a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) $2\sqrt{2}$ (e) 2
86. The unit vector in the direction of the vector $\overrightarrow{AB}$ if $A = (-2, -1, 3)$ and $B = (1, 1, 0)$ is $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, then $\alpha + \beta$ is
 (a) $\frac{3}{\sqrt{22}}$ (b) $\frac{5}{\sqrt{22}}$ (c) $\frac{-3}{\sqrt{22}}$ (d) $\frac{-5}{\sqrt{22}}$ (e) $\frac{2}{\sqrt{22}}$
87. If $\begin{pmatrix} 3x-y & x+3y \\ 2x-z & 2y+z \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 5 & 5 \end{pmatrix}$, then $x + y + z$ equals
 (a) 3 (b) 6 (c) 9 (d) 12 (e) 11
88. If the product $abc = 1$, then the value of the determinant $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ is
 (a) 1 (b) 2 (c) 3 (d) 4 (e) 5
89. If (x, y, z) is the solution of the equations
 $4x + y = 7$
 $3y + 4z = 5$
 $5x + 3z = 2$
 Then the value of $x + y + z$ equals
 (a) 8 (b) 6 (c) 3 (d) 0 (e) 1
90. If $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$ is the inverse of the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $ad - bc = 1$, then g equals
 (a) c (b) $-c$ (c) b (d) $-b$ (e) d
91. If $f: R \rightarrow R$ is a function defined by $f(x) = x^2$, then which of the following is true?
 (a) f is 1-1 but not onto
 (b) f is onto but not 1-1
 (c) f is neither 1-1 nor onto
 (d) f is both 1-1 and onto
 (e) $f^{-1}: R \rightarrow R$ exists
92. Consider the set $A = \{1, 2, 3\}$ along with the relation $R = \{(1, 1), (2, 2), (1, 2), (2, 1), (3, 3)\}$. Which of the following statements is true?
 (a) The relation is symmetric but not transitive
 (b) The relation is transitive but not symmetric
 (c) The relation is neither symmetric nor transitive
 (d) The relation is both symmetric and transitive
 (e) The relation is a function
93. If $(-\sqrt{3} - i)^{30} = -4^k$, then the value of k is
 (a) 15 (b) 20 (c) 25 (d) 30 (e) 60
94. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ is equal to
 (a) 128ω (b) -128ω
 (c) $128\omega^2$ (d) $-128\omega^3$
 (e) $-128\omega^2$
95. The value of $\left[\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right]^4$ is
 (a) $-i\pi$ (b) $i\pi$ (c) i (d) $-i$ (e) π
96. If $\arg(z_1) = \arg(z_2)$, then
 (a) $z_2 = kz_1^{-1}$, ($k > 0$) (b) $z_2 = kz_1$, ($k > 0$)
 (c) $|z_2| = |\bar{z}_1|$ (d) $z_1 = z_2$
 (e) $|z_2| = |z_1|$
97. The value of $\tan \left[\sin^{-1} \frac{5}{13} + \cot^{-1} \frac{4}{3} \right]$ is
 (a) $\frac{26}{11}$ (b) $\frac{56}{33}$ (c) $\frac{63}{41}$ (d) $\frac{65}{43}$ (e) $\frac{32}{13}$
98. If $\tan^{-1} x + 2\cot^{-1} x = \frac{\pi}{3}$, then the value of x is
 (a) $-\sqrt{3}$ (b) $-\sqrt{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$ (e) $\sqrt{5}$
99. Which of the following is not a solution of the equation $3 \tan^2 \theta - \sin \theta = 0$?
 (a) $n\pi$ (b) $n\frac{\pi}{2}$
 (c) $n\pi + (-1)^n \frac{\pi}{6}$ (d) 0
 (e) π
100. If $\sqrt{\frac{y}{x}} + \sqrt{\frac{x}{y}} = 1$, then $\frac{dy}{dx}$ equals



(a) $\sqrt{\frac{y}{x}}$ (b) $\sqrt{\frac{x}{y}}$ (c) $\frac{y}{x}$ (d) $\frac{x}{y}$ (e) xy

101. If $x = \frac{3t}{1+t^3}$ and $y = \frac{3t^2}{1+t^3}$, then $\frac{dy}{dx}$ at $t = 1$ equals

(a) -6 (b) -1 (c) 1 (d) 6 (e) 4

102. The equation of the normal to the curve given by $x^2 + 2x - 3y + 3 = 0$ at the point (1, 2) is

(a) $3x + 4y - 11 = 0$ (b) $3x - 4y + 11 = 0$
(c) $-3x + 4y - 11 = 0$ (d) $3x - 4y - 11 = 0$
(e) $-3x - 4y - 11 = 0$

103. A point of inflection of the curve given by $y = x^3 - 6x^2 + 12x + 50$ occurs when

(a) $x = 2/3$ (b) $x = 3/2$
(c) $x = 2$ (d) $x = 3$
(e) $x = 0$

104. The value of the integral $\int_0^{\pi/2} \log \tan \theta \, d\theta$ is

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\log 2$ (e) 2

105. The area enclosed between the curve $y = 11x - 24 - x^2$ and the line $y = x$ is

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{4}{3}$ (e) $\frac{1}{2}$

106. The solution of the differential equation $\frac{dy}{dx} = \frac{y^2}{x}$ passing through the point (1, -1) is

(a) $\frac{1}{y} + \log x = 0$ (b) $\frac{1}{y} - \log x = 0$
(c) $y + \log x = 0$ (d) $y - \log x = 0$
(e) $y \log x = 0$

107. The maxima and minima of the function $2x^3 - 15x^2 + 36x + 10$ occur respectively at

(a) $x = 1, x = 3$ (b) $x = 2, x = 1$
(c) $x = 3, x = 2$ (d) $x = 1, x = 2$
(e) $x = 2, x = 3$

108. In a class of 100 students, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?

(a) 73 (b) 85 (c) 68 (d) 74 (e) 65

109. Let $x_1, x_2, \dots, x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n is

(a) 15 (b) 10 (c) 9 (d) 12 (e) 18

110. If M and N are events such that $P(M \cup N) = \frac{3}{4}$,

$P(M \cap N) = \frac{1}{4}$, $P(\bar{M}) = \frac{2}{3}$, then $P(\bar{M} \cap N)$ is

(a) $\frac{15}{12}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$ (e) $\frac{5}{12}$

111. Cards marked with numbers 2 to 105 are placed in a box and mixed. One card is chosen at random. The probability that the number on the card is less than 15 is

(a) $\frac{1}{8}$ (b) $\frac{1}{9}$ (c) $\frac{7}{8}$ (d) $\frac{8}{9}$ (e) $\frac{2}{7}$

112. An urn contains 4 black, 5 white and 6 red balls. One ball is drawn at random. The probability that it is not black is

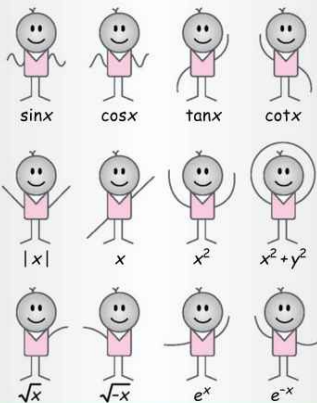
(a) $\frac{4}{15}$ (b) $\frac{9}{15}$ (c) $\frac{11}{15}$ (d) $\frac{13}{15}$ (e) $\frac{14}{15}$

113. In a chess tournament, assume that your probability of winning a game is 0.3 against level 1 players, 0.4 against level 2 players and 0.5 against level 3



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players. It is further assumed that among the players 50% are at level 1, 25% are at level 2 and the remaining are at level 3. The probability of winning a game against a randomly chosen player is

- (a) 0.275 (b) 0.375
(c) 0.225 (d) 0.325
(e) 0.125

114. A man repays a loan of ₹ 3250 by paying ₹ 20 in the first month and then increases the payment by ₹ 15 every month. The number of months it takes to clear the loan is

- (a) 20 (b) 25 (c) 35 (d) 40 (e) 10

115. The coefficient of x^3 in the expansion of $\left(x^2 - \frac{2}{x}\right)^6$ is

- (a) -160 (b) -80 (c) -40 (d) 0 (e) -10

116. If the equation of the sphere through the circle $x^2 + y^2 + z^2 = 5$; $2x + 3y + 4z = 5$ and through the origin is $x^2 + y^2 + z^2 - 2x - 3y - 4z + C = 0$, then the value of C is

- (a) 1 (b) -1 (c) 0 (d) 5 (e) 2

117. The equation of the plane containing the lines

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} \text{ and } \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} \text{ is}$$

- (a) $x + 2y + z = 0$ (b) $x - 2y + z = 0$
(c) $x - 2y - z = 0$ (d) $x + 2y - z = 0$
(e) $2y - x - z = 0$

118. A value of c for which the conclusion of mean value theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is

- (a) $8\log_3 e$ (b) $\frac{1}{2}\log_3 3$
(c) $\log_3 e$ (d) $\log_3 3$
(e) $2\log_3 e$

119. From 4 men and 6 ladies a committee of five is to be selected. The number of ways in which the committee can be formed so that men are in majority is

- (a) 68 (b) 156 (c) 60 (d) 72 (e) 66

120. The degree of the differential equation

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = l \frac{d^2 y}{dx^2} \text{ is}$$

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

SOLUTIONS

1. (c) : The given equation is $x^2 + 6x + 4y + 5 = 0$
 $\Rightarrow x^2 + 6x + 9 - 9 = -4y - 5$
 $\Rightarrow (x + 3)^2 = -4y + 4 \Rightarrow (x + 3)^2 = -4(y - 1)$
 $\therefore$ The axis of parabola is $x + 3 = 0$

2. (b) : The given equation of ellipse is

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 1$$

$$\text{Now, } e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$$

$$\therefore \text{Distance between foci} = 2ae = 2 \times 3 \times \frac{\sqrt{5}}{3} = 2\sqrt{5}.$$

3. (b) : The given equation of circles are
 $2x^2 + 2y^2 - 4x + 6y = 3$ and $x^2 + y^2 + kx + y = 0$
 Now, the two circles cut orthogonally if

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2 \Rightarrow 2(-1)\left(\frac{k}{2}\right) + 2\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = \frac{-3}{2}$$

$$\Rightarrow -k = \frac{-6}{2} \Rightarrow k = 3.$$

4. (e) : Since the circle touches the x -axis at $(3, 0)$.
 $\therefore$ Its center be $C = (3, \pm r)$

Now, point $P(1, -2)$ also lies on circle

$$\therefore (PC)^2 = r^2$$

$$\Rightarrow (3-1)^2 + (\pm r + 2)^2 = r^2 \Rightarrow \pm 4r = -8$$

$$\Rightarrow r = \pm 2$$

$\therefore$ Centre is $C(3, \pm 2)$

So, the equation of the circle will be

$$(x-3)^2 + (y \pm 2)^2 = 4$$

Since $P(1, -2)$ does not satisfy $(x-3)^2 + (y-2)^2 = 4$

$$\therefore \text{Equation of circle is } (x-3)^2 + (y+2)^2 = 2^2 \quad \dots(i)$$

Thus, only $(5, -2)$ satisfy equation (i).

5. (e) : Given, α and β are roots of the equation

$$x^2 + \alpha x + \beta = 0$$

$$\therefore \alpha^2 + \alpha^2 + \beta = 0 \Rightarrow \beta = -2\alpha^2 \quad \dots(i)$$

$$\text{and } \beta^2 + \alpha\beta + \beta = 0 \quad \dots(ii)$$

Using (i) in (ii), we get $(-2\alpha^2)^2 + \alpha(-2\alpha^2) + (-2\alpha^2) = 0$

$$\Rightarrow 4\alpha^4 - 2\alpha^3 - 2\alpha^2 = 0$$

$$\Rightarrow 2\alpha^2(2\alpha^2 - \alpha - 1) = 0 \Rightarrow 2\alpha^2(\alpha - 1)(2\alpha + 1) = 0$$

$$\Rightarrow \alpha = 0, \alpha = 1, \alpha = -\frac{1}{2}$$

$$\text{When } \alpha = 0, \beta = 0; \alpha = 1, \beta = -2; \alpha = -\frac{1}{2}, \beta = -\frac{1}{2}$$

But, $\alpha = 0, \beta = 0$ and $\alpha = -\frac{1}{2}, \beta = -\frac{1}{2}$ does not satisfy

given equation.

Thus, $\alpha = 1, \beta = -2$





6. (c) : Given $\vec{a} = (1, 1, -1)$, $\vec{b} = (-1, 2, 1)$ and $\vec{c} = (-1, 2, -1)$

Now, $\vec{a} + \vec{b} = (1-1)\hat{i} + (1+2)\hat{j} + (-1+1)\hat{k} = 3\hat{j}$

$\vec{b} + \vec{c} = (-1-1)\hat{i} + (2+2)\hat{j} + (1-1)\hat{k} = -2\hat{i} + 4\hat{j}$

$$\text{So, } (\vec{a} + \vec{b}) \times (\vec{b} + \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ -2 & 4 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(6) = 6\hat{k}$$

$$\therefore |(\vec{a} + \vec{b}) \times (\vec{b} + \vec{c})| = 6.$$

7. (b) : Given a particle is displaced from the point $P(2, 1, -1)$ to point $Q(4, 3, -4)$.

$$\therefore \vec{PQ} = (4-2)\hat{i} + (3-1)\hat{j} + (-4+1)\hat{k} \\ = 2\hat{i} + 2\hat{j} - 3\hat{k} = \vec{d} \text{ (say)}$$

Now, work done by the force $= \vec{F} \cdot \vec{d}$
 $= (2\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 4 + 8 + 15 = 27.$

8. (b) : Let $\vec{a} = 4\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{b} = m\hat{i} - 4\hat{j} + \hat{k}$
 Since $\vec{a}$ and $\vec{b}$ are perpendicular.

$$\therefore \vec{a} \cdot \vec{b} = 0 \Rightarrow (4\hat{i} - 3\hat{j} + 5\hat{k}) \cdot (m\hat{i} - 4\hat{j} + \hat{k}) = 0 \\ \Rightarrow 4m + 12 + 5 = 0 \Rightarrow 4m = -17 \Rightarrow m = -\frac{17}{4}.$$

9. (None of the options is correct) :

$$\text{Given, } 3A + B = \begin{bmatrix} 9 & 11 & 3 \\ 12 & 14 & 19 \end{bmatrix} \quad \dots(i)$$

$$2A - 3B = \begin{bmatrix} -16 & 11 & 2 \\ -3 & -22 & 9 \end{bmatrix} \quad \dots(ii)$$

Now multiplying (i) by 2 and (ii) by 3, we get

$$6A + 2B = \begin{bmatrix} 18 & 22 & 6 \\ 24 & 28 & 38 \end{bmatrix} \quad \dots(iii)$$

$$\text{and } 6A - 9B = \begin{bmatrix} -48 & 33 & 6 \\ -9 & -66 & 27 \end{bmatrix} \quad \dots(iv)$$

$$\text{Subtracting (iv) from (iii), we get } 11B = \begin{bmatrix} 66 & -11 & 0 \\ 33 & 94 & 11 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 6 & -1 & 0 \\ 3 & 94/11 & 1 \end{bmatrix}$$

$$10. (a) : \text{Given, } \begin{bmatrix} a^3 + 1 & a^2 & a \\ b^3 + 1 & b^2 & b \\ c^3 + 1 & c^2 & c \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} a^3 & a^2 & a \\ b^3 & b^2 & b \\ c^3 & c^2 & c \end{bmatrix} \begin{bmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{bmatrix} = 0$$

$$\Rightarrow abc \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} + \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1 + abc) \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1 + abc)[a^2(b-c) - a(b^2 - c^2) + 1(b^2c - bc^2)] = 0$$

$$\Rightarrow (1 + abc)[(b-c)(a^2 - a(b+c) + bc)] = 0$$

$$\Rightarrow (1 + abc)(b-c)[a(a-b) - c(a-b)] = 0$$

$$\Rightarrow (1 + abc)(b-c)(a-b)(a-c) = 0$$

$$\Rightarrow 1 + abc = 0$$

$$\Rightarrow abc = -1 \quad [\because a, b \text{ and } c \text{ are distinct}]$$

11. (c) : We have, $AX = B$

$$\text{where } A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 4 \\ 4 & -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\text{Now, } |A| = 1(-2+4) + 1(-4-16) - 2(-2-4) \\ = 2 - 20 + 12 = -6 \neq 0$$

Let A_{ij} be the cofactor of element of a_{ij} in A .

$$\therefore A_{11} = 2, A_{12} = 20, A_{13} = -6$$

$$A_{21} = 0, A_{22} = 6, A_{23} = -3$$

$$A_{31} = -2, A_{32} = -8, A_{33} = 3$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & 20 & -6 \\ 0 & 6 & -3 \\ -2 & -8 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 \\ 20 & 6 & -8 \\ -6 & -3 & 3 \end{bmatrix}$$

$$\therefore \text{Now, } A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{-1}{6} \begin{bmatrix} 2 & 0 & -2 \\ 20 & 6 & -8 \\ -6 & -3 & 3 \end{bmatrix}$$

Hence, the solution of the equation is $X = A^{-1}B$

$$\text{Thus, } X = \frac{-1}{6} \begin{bmatrix} 2 & 0 & -2 \\ 20 & 6 & -8 \\ -6 & -3 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix} = \frac{-1}{6} \begin{bmatrix} -16 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 8/3 \\ -1/3 \\ 0 \end{bmatrix}$$

$$\therefore y = -1/3.$$

$$12. (a) : \text{Let } A = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} \therefore A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad [\text{Given}]$$

$$\text{Now } AA^{-1} = I \Rightarrow \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+5c & b+5d \\ 7a-3c & 7b-3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Now } b+5d = 0 \dots(i) \text{ and } 7b-3d = 1 \dots(ii)$$

$$\text{Solving (i) and (ii), we get } d = \frac{-1}{38}.$$



13. (d) : Here $f: R \rightarrow R$ is a function defined by $f(x) = \sin x$

Since $f(0) = f(\pi)$

So, f is not one-one.

Now, $4 \in R$ but $f(x) \neq 4$ for any $x \in R$

$\therefore f$ is not onto.

$\therefore f$ is neither one - one nor onto.

14. (d) : Given $M = \{1, 2, 3\}$ and $R = \{(1, 2), (1, 1), (3, 1), (3, 4), (3, 3), (4, 3)\}$.

Now R is not reflexive, as R does not contain $(2, 2)$.

R is not symmetric since R contains $(1, 2)$ but not $(2, 1)$.

R is not transitive since R contains $(3, 1)$ and $(1, 2)$ but not $(3, 2)$.

15. (b) : Here $z_1 = 1 + i$ and $z_2 = 1 + i$

$$\text{Now, } \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2 = \arg z_1 + \arg z_2 \quad [\because \arg \bar{z} = -\arg z]$$

$$= \arg(1 + i\sqrt{3}) + \arg(1 + i) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) + \tan^{-1}\left(\frac{1}{1}\right)$$

$$= \frac{\pi}{3} + \frac{\pi}{4} = \frac{4\pi + 3\pi}{12} = \frac{7\pi}{12}$$

$$16. (d) : \sqrt{2} \left[\sin \frac{\pi}{8} + i \cos \frac{\pi}{8} \right]^6$$

$$= \sqrt{2} i^6 \left[\cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right]^6 = -\sqrt{2} \left[\cos \frac{\pi}{8} - i \sin \frac{\pi}{8} \right]^6$$

$$= -\sqrt{2} \left[\cos \frac{6\pi}{8} - i \sin \frac{6\pi}{8} \right] = -\sqrt{2} \left[\cos \frac{3\pi}{4} - i \sin \frac{3\pi}{4} \right]$$

$$= -\sqrt{2} \left[\cos \left(\pi - \frac{\pi}{4} \right) - i \sin \left(\pi - \frac{\pi}{4} \right) \right]$$

$$= -\sqrt{2} \left[-\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right] = -\sqrt{2} \left(\frac{-1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right) = 1 + i$$

17. (e) : Given $z^2 + z + 1 = 0 \Rightarrow z = \omega, \omega^2$

If $z = \omega$, then

$$\left(z + \frac{1}{z} \right)^2 + \left(z^2 + \frac{1}{z^2} \right)^2 + \left(z^3 + \frac{1}{z^3} \right)^2 + \dots + \left(z^6 + \frac{1}{z^6} \right)^2$$

$$= \left(\omega + \frac{1}{\omega} \right)^2 + \left(\omega^2 + \frac{1}{\omega^2} \right)^2 + \dots + \left(\omega^6 + \frac{1}{\omega^6} \right)^2$$

$$= \left(\omega + \frac{1}{\omega} \right)^2 + \left(\omega^2 + \frac{1}{\omega^2} \right)^2 + (1+1)^2 + \left(\omega + \frac{1}{\omega} \right)^2$$

$$+ \left(\omega^2 + \frac{1}{\omega^2} \right)^2 + (1+1)^2$$

$$= 2 \left(\omega + \frac{1}{\omega} \right)^2 + 2 \left(\omega^2 + \frac{1}{\omega^2} \right)^2 + 4 + 4$$

$$= 2 \left[(\omega + \omega^2)^2 + (\omega^2 + \omega)^2 \right] + 8$$

$$= 4 (\omega + \omega^2)^2 + 8 = 4 (-1)^2 + 8 = 12$$

Similarly, if $z = \omega^2$, then required sum is 12.

$$18. (a) : \text{We have, } \tan \left[\sin^{-1} \left(\frac{-1}{\sqrt{2}} \right) \right] = \tan \left[-\sin^{-1} \left(\frac{1}{\sqrt{2}} \right) \right]$$

$$= \tan \left(-\frac{\pi}{4} \right) = -\tan \left(\frac{\pi}{4} \right) = -1.$$

$$19. (a) : \text{We have, } \sin^{-1} x + \cos^{-1} 2x = \frac{\pi}{6}$$

$$\Rightarrow \cos^{-1} 2x = \frac{\pi}{6} - \sin^{-1} x \Rightarrow 2x = \cos \left(\frac{\pi}{6} - \sin^{-1} x \right)$$

$$\Rightarrow 2x = \cos \frac{\pi}{6} \cos(\sin^{-1} x) + \sin \frac{\pi}{6} \sin(\sin^{-1} x)$$

$$= \frac{\sqrt{3}}{2} \cos(\cos^{-1} \sqrt{1-x^2}) + \frac{x}{2}$$

$$\Rightarrow 2x - \frac{x}{2} = \frac{\sqrt{3}}{2} \sqrt{1-x^2} \Rightarrow \frac{3x}{2} = \frac{\sqrt{3}}{2} \sqrt{1-x^2}$$

$$\Rightarrow \sqrt{3}x = \sqrt{1-x^2} \Rightarrow 3x^2 = 1 - x^2$$

$$\Rightarrow 4x^2 = 1 \Rightarrow x^2 = \frac{1}{4} \Rightarrow x = \pm \frac{1}{2}$$

20. (a) : Here $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$

$$\therefore \frac{dx}{dt} = -2 \sin t + 2 \sin 2t, \frac{dy}{dt} = 2 \cos t - 2 \cos 2t$$

$$\text{Now, } \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2(\cos t - \cos 2t)}{-2(\sin t - \sin 2t)} = -\left(\frac{\cos t - \cos 2t}{\sin t - \sin 2t} \right)$$

$$\therefore \left(\frac{dy}{dx} \right)_{t=\pi/2} = -\left(\frac{0 - (-1)}{1 - 0} \right) = -1$$

21. (b) : The given curve is $x^2 + 2x - 3y + 3 = 0 \dots (i)$

Differentiating (i) w.r.t. x , we get

$$2x + 2 - \frac{3dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1}{3}(2x + 2) \Rightarrow \left(\frac{dy}{dx} \right)_{(1,2)} = \frac{4}{3}$$

Now, equation of tangent at $(1, 2)$ is

$$(y - 2) = \frac{4}{3}(x - 1) \Rightarrow 3y - 6 = 4x - 4$$

$$\Rightarrow 3y - 4x - 2 = 0.$$

$$22. (e) : \lim_{x \rightarrow \infty} \frac{x^3 \sin(1/x) - 2x^2}{1 + 3x^2} \quad \left(\frac{\infty}{\infty} \text{ form} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{3x^2 \sin\left(\frac{1}{x}\right) + x^3 \cos\left(\frac{1}{x}\right) \left(\frac{-1}{x^2} \right) - 4x}{6x}$$

[Applying L' Hospital's rule]



$$\begin{aligned}
 &= \lim_{x \rightarrow \infty} \frac{3x^3 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) - 4x}{6x} \quad \left(\frac{\infty}{\infty} \text{ form}\right) \\
 &= \lim_{x \rightarrow \infty} \frac{6x \sin\left(\frac{1}{x}\right) - 3 \cos\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right) - \frac{1}{x} \sin\left(\frac{1}{x}\right) - 4}{6} \\
 &= \frac{6 - 3 - 1 - 4}{6} = \frac{-1}{3} \quad [\text{Applying L' Hospital's rule}]
 \end{aligned}$$

23. (a) : The given curve is $y = \left(\frac{1}{x}\right)^x$

Taking log on both sides, we get

$$\log y = x \log\left(\frac{1}{x}\right) = -x \log x$$

Differentiating (i), w.r.t. x , we get

$$\frac{1}{y} \frac{dy}{dx} = -\log x - x \cdot \frac{1}{x} \Rightarrow \frac{dy}{dx} = -y(\log x + 1)$$

Putting $\frac{dy}{dx} = 0$, we get $-y(\log x + 1) = 0 \Rightarrow \log x + 1 = 0$

$$\Rightarrow \log x = -1 \Rightarrow x = e^{-1}$$

$$\text{Also, } \frac{d^2y}{dx^2} = -\left[\frac{dy}{dx}(\log x + 1) + y\left(\frac{1}{x}\right)\right] < 0 \text{ for } x = e^{-1}$$

$\therefore f(x)$ is maximum at $x = e^{-1}$

$$\therefore \text{Maximum value of } y \text{ is } \left(\frac{1}{e^{-1}}\right)^{e^{-1}} = (e)^{\frac{1}{e}}$$

$$24. (a) : \text{Let } I = \int_0^{\pi} \frac{\cos x}{1 + \sin^2 x} dx \quad \dots(i)$$

$$\text{Also, } I = \int_0^{\pi} \frac{\cos(\pi - x)}{1 + \sin^2(\pi - x)} dx = \int_0^{\pi} \frac{-\cos x}{1 + \sin^2 x} dx \quad \dots(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{\cos x - \cos x}{1 + \sin^2 x} dx = 0 \Rightarrow I = 0.$$

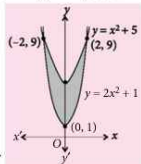
25. (d) : The given curves are $y = 2x^2 + 1$ and $y = x^2 + 5$ $\dots(i)$
 $\dots(ii)$

Solving (i) and (ii), we get $x = \pm 2, y = 9$

So, the point of intersection of (i) and (ii) are (2, 9) and (-2, 9).

$\therefore$ Required area

$$\begin{aligned}
 &= \int_{-2}^2 (x^2 + 5 - 2x^2 - 1) dx \\
 &= \int_{-2}^2 (4 - x^2) dx = \left[4x - \frac{x^3}{3}\right]_{-2}^2 \\
 &= \left(8 - \frac{8}{3}\right) + \left(8 - \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3}
 \end{aligned}$$



26. (b) : The given differential equation is $5y dx = 2x dy$

$\Rightarrow 5 \frac{dx}{x} = 2 \frac{dy}{y}$. Integrating both sides, we get

$$5 \int \frac{dx}{x} = 2 \int \frac{dy}{y} \Rightarrow 5 \log x = 2 \log y + \log c \quad \dots(i)$$

Since, curve passes through (1, 1), so $\log c = 0$

$$\Rightarrow c = 1$$

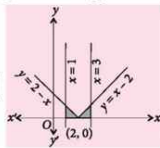
Thus, from (i), $5 \log x = 2 \log y$

$$27. (e) : \text{The required area} = \int_1^2 (-x+2) dx + \int_2^3 (x-2) dx$$

$$= \left[-\frac{x^2}{2} + 2x\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= \left(-\frac{4}{2} + 4\right) - \left(-\frac{1}{2} + 2\right) + \left(\frac{9}{2} - 6\right)$$

$$= \left(-\frac{4}{2} - 4\right) = 2 - \frac{3}{2} - \frac{3}{2} + 2 = 1$$



28. (e) : Given, mean = 21 and median = 22.

Since, mode = 3 Median - 2 Mean

$$\therefore \text{Mode} = 3(22) - 2(21) = 66 - 42 = 24.$$

**SAMURAI
SUDOKU**

ANSWER • MAY 2019



2	7	6	4	8	5	9	3	1
5	8	3	1	9	6	2	4	7
1	9	4	2	3	7	6	5	8
3	5	1	9	2	4	8	7	6
8	4	2	7	6	1	3	9	5
7	6	9	8	5	3	4	1	2
4	2	5	3	7	8	1	6	9
9	1	7	6	4	2	5	8	3
6	3	8	5	1	9	7	2	4

	2	4	5	6	8	9	3	7	1
	3	1	6	7	4	2	9	5	8
	8	9	7	3	1	5	4	6	2

8	7	3	1	9	6	4	5	2	1	7	8	6	9	3	8	4	1	7	5	2
2	9	4	7	8	5	6	3	1	9	5	4	2	8	7	9	3	5	4	1	6
1	6	5	2	3	4	9	7	8	2	6	3	1	4	5	6	2	7	3	8	9
6	3	2	9	4	1	7	8	5				8	5	1	2	6	4	9	7	3
9	5	8	3	2	7	1	6	4				4	7	9	5	8	3	2	6	1
7	4	1	5	6	8	2	9	3				3	2	6	7	1	9	5	4	8
4	1	6	8	5	9	3	2	7				7	1	2	3	5	8	6	9	4
3	8	9	4	7	2	5	1	6				5	6	4	1	9	2	8	3	7
5	2	7	6	1	3	8	4	9				9	3	8	4	7	6	1	2	5



29. (e) : Given $\sigma_x^2 = 4$ and $\sigma_y^2 = 5$

$$\text{Also } \bar{x} = \frac{\sum x_i}{5} = 2 \text{ and } \bar{y} = \frac{\sum y_i}{5} = 4$$

$$\therefore \sum x_i = 10 \text{ and } \sum y_i = 20$$

$$\text{Now, } \sigma_x^2 = \frac{1}{5} \sum x_i^2 - (\bar{x})^2 \text{ and } \sigma_y^2 = \frac{1}{5} \sum y_i^2 - (\bar{y})^2$$

$$\Rightarrow \sigma_x^2 = \frac{1}{5} \sum x_i^2 - (2)^2 \text{ and } \sigma_y^2 = \frac{1}{5} \sum y_i^2 - (4)^2$$

$$\Rightarrow 4 = \frac{1}{5} \sum x_i^2 - 4 \text{ and } 5 = \frac{1}{5} \sum y_i^2 - 16$$

$$\sum x_i^2 = 40 \text{ and } \sum y_i^2 = 105$$

$$\therefore \text{ Combined variance} = \frac{1}{n_1 + n_2} (\sum x_i^2 + \sum y_i^2) - \left(\frac{\bar{x} + \bar{y}}{2} \right)^2$$

$$= \frac{1}{10} (40 + 105) - \left(\frac{6}{2} \right)^2 = \frac{145}{10} - 9 = \frac{11}{2}$$

30. (d) : Mean of first n odd numbers = $\frac{n^2}{81}$ [Given]

$$\Rightarrow \frac{\text{Sum of first } n \text{ odd numbers}}{n} = \frac{n^2}{81}$$

$$\Rightarrow \frac{n^2}{n} = \frac{n^2}{81} \Rightarrow n = 81$$

31. (a) : Let the bag contains x blue balls. According to question, $P(\text{blue balls}) = 2 P(\text{red balls})$

$$\Rightarrow \frac{x}{5+x} = 2 \left(\frac{5}{5+x} \right) \Rightarrow x = 10$$

32. (b) : In throwing 2 dice together, the number of all possible outcomes = $6 \times 6 = 36 = n(S)$

Let E be the event of getting a total of 8.

$$\therefore E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\} \Rightarrow n(E) = 5$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

33. (b) : Let A_1 be the event of playing with opponent of type i .

$$\therefore P(A_1) = \frac{50}{100}, P(A_2) = \frac{25}{100}, P(A_3) = \frac{25}{100}$$

Let E denotes the probability of winning the game.

$$\text{Then, } P(E/A_1) = 0.3, P(E/A_2) = 0.4, P(E/A_3) = 0.5$$

$\therefore$ By law of total probability, we have

$$P(E) = P(A_1) P(E/A_1) + P(A_2) P(E/A_2) + P(A_3) P(E/A_3)$$

$$= \frac{50}{100} \times 0.3 + \frac{25}{100} \times 0.4 + \frac{25}{100} \times 0.5 = 0.375$$

$$\text{Required probability} = \frac{\frac{50}{100} \times 0.3}{0.375} = 0.4$$

...(i)

34. (b) : Let the value of the first prize = x

Then the value of the second prize = $x - 20$

Value of the third prize = $(x - 20) - 20 = x - 40$

Value of the fourth prize = $(x - 40) - 20 = x - 60$

Now, according to question

$$x + x - 20 + x - 40 + x - 60 = 280$$

$$\Rightarrow 4x - 120 = 280$$

$$\Rightarrow 4x = 400 \Rightarrow x = 100$$

Thus, value of the fourth prize = $100 - 60 = 40$.

$$\begin{aligned} 35. (c) : \text{ We have, } (1+x+2x^2)(1-2x)^5 \\ = (1+x+2x^2)[1-5(2x)+10(4x^2)-(10)(8x^3) \\ +5(16x^4)-32x^5] \end{aligned}$$

$$\therefore \text{ The coefficient of } x^3 = -80 + 40 - 20 = -60$$

$$36. (c) : \text{ We have, } \left(x^2 - \frac{2}{x} \right)^6$$

$$\therefore T_{r+1} = {}^6C_r (x^2)^{6-r} \left(-\frac{2}{x} \right)^r = {}^6C_r x^{2r-6r} (-2)^{6-r}$$

$$= {}^6C_r x^{3r-6} (-2)^{6-r}$$

For constant term, $3r - 6 = 0 \Rightarrow r = 2$

$$\therefore T_{2+1} = T_3 = {}^6C_2 (-2)^4 = 240$$

Thus, constant term is 240.

37. (b) : Equation of the sphere passing through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ is

$$(x^2 + y^2 + z^2 - 9) + \lambda(2x + 3y + 4z - 5) = 0 \quad \dots(i)$$

Since, it passes through $(1, 2, 3)$

$$\therefore (1 + 4 + 9 - 9) + \lambda(2 + 6 + 12 - 5) = 0$$

$$\Rightarrow 5 + 15\lambda = 0 \Rightarrow \lambda = -\frac{1}{3}$$

$$\text{From (i), } 3(x^2 + y^2 + z^2 - 9) - (2x + 3y + 4z - 5) = 0$$

$$\Rightarrow 3(x^2 + y^2 + z^2) - 2x - 3y - 4z = 22$$

$$\therefore C = 22$$

38. (e)

39. (e) : Since, $f(x)$ is differentiable in $[0, 1]$ and hence continuous in $[0, 1]$.

$\therefore$ There exists $c \in (0, 1)$ such that

$$f'(c) = \frac{f(1) - f(0)}{1 - 0}$$

$$\Rightarrow f'(c) = \frac{6-2}{1} = 4$$

$$\text{Now, } f'(c) = 2g'(c) \quad (\text{Given})$$

$$\Rightarrow g'(c) = 2$$

Similarly, $g(x)$ satisfies all the conditions of Lagrange's mean value theorem.

$$\therefore \text{ There exists } c \in (0, 1) \text{ such that } g'(c) = \frac{g(1) - g(0)}{1 - 0}$$

$$\Rightarrow 2 = \frac{g(1) - 0}{1} \Rightarrow g(1) = 2$$





40. (c) : The equation of the curve is $y = x + \frac{4}{x^2}$... (i)

Differentiating (i), w.r.t. x , we get

$$\frac{dy}{dx} = 1 + \frac{4(-2)}{x^3} = 1 - \frac{8}{x^3}$$

Since, the curve is parallel to x -axis.

$$\therefore \frac{dy}{dx} = 0 \Rightarrow 1 - \frac{8}{x^3} = 0 \Rightarrow 1 = \frac{8}{x^3}$$

$$\Rightarrow x^3 = 8 \Rightarrow x = 2.$$

Putting $x = 2$ in (i), we get $y = 2 + \frac{4}{4} = 3$

$\therefore$ Equation of tangent at $(2, 3)$ is $(y - 3) = 0(x - 2)$

$$\Rightarrow y = 3.$$

$$\begin{aligned} 41. (c) : \left(x^2 + \frac{3}{x}\right)^4 &= {}^4C_0 \left(\frac{3}{x}\right)^4 + {}^4C_1 x^2 \left(\frac{3}{x}\right)^3 + {}^4C_2 x^4 \left(\frac{3}{x}\right)^2 \\ &\quad + {}^4C_3 x^6 \left(\frac{3}{x}\right)^1 + {}^4C_4 x^8 \left(\frac{3}{x}\right)^0 \\ &= \frac{81}{x^4} + 4 \times x^2 \times \frac{27}{x^3} + 6 \times 9 \times x^2 + 12x^5 + x^8 \\ &= \frac{81}{x^4} + \frac{108}{x} + 54x^2 + 12x^5 + x^8 \end{aligned}$$

Here, 81 is coefficient of x^{-4} .
 $\therefore k = -4$.

42. (c) : Let us fix K at the extreme left position, then we count the arrangements of the remaining 6 letters with A occurring twice.

$\therefore$ The required number of words starting with K are $\frac{6!}{2!} = 360$.

43. (b) : Since the bag contains 3 black and 2 white balls.

$$\therefore P(B) = \frac{3}{5} \text{ and } P(W) = \frac{2}{5}$$

Case I : When black ball is drawn and is put back in the bag along with one ball of black colour.

$$\therefore P(B) = \frac{4}{5} \text{ and } P(W) = \frac{2}{5}$$

Case II : When white ball is drawn and is put back in the bag along with one ball of white colour.

$$\therefore P(B) = \frac{3}{5} \text{ and } P(W) = \frac{3}{5}$$

$$\therefore \text{Required probability} = \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$$

$$= \frac{6}{30} + \frac{6}{30} = \frac{12}{30} = \frac{2}{5}$$

44. (b) : Since it is given that $P(A \cup B) = P(A \cap B)$

$$\Rightarrow P(A) = P(B) \Rightarrow P(B) = 1/3 \quad [\text{Given } P(A) = \frac{1}{3}]$$

45. (c) : When 3 dice are rolled together, the number of all possible outcomes = $6 \times 6 \times 6 = 216 = n(S)$

Let E be the event of getting same number on each of them.

$$\therefore E = \{(1, 1, 1), (2, 2, 2), (3, 3, 3), (4, 4, 4), (5, 5, 5), (6, 6, 6)\} \Rightarrow n(E) = 6$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{216} = \frac{1}{36}$$

46. (d) : Since $1 + \omega + \omega^2 = 0 \Rightarrow 1 + \omega = -\omega^2$... (i)
 $\therefore (1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2 = 1 + \omega \quad \therefore a = 1$

47. (b) : Here $w = \frac{1-iz}{z-i}$

$$\text{Now, } |w| = 1 \Rightarrow \left| \frac{1-iz}{z-i} \right| = 1$$

$$\Rightarrow \left| \frac{1-i^2-z}{z-i} \right| = 1 \Rightarrow |(-i)(i+z)| = |z-i|$$

$$\Rightarrow |(-i)(i+x+iy)| = |x+iy-i| \quad [\text{Putting } z = x+iy]$$

$$\Rightarrow |1+y-ix| = |x+(y-1)i|$$

$$\Rightarrow \sqrt{(1+y)^2 + x^2} = \sqrt{x^2 + (y-1)^2}$$

Squaring both sides, we get

$$y^2 + 1 + 2y = y^2 + 1 - 2y$$

$$\Rightarrow 4y = 0 \Rightarrow y = 0.$$

$$\therefore z = x$$

So, if $|w| = 1$, then z lies on real axis.

48. (b) : Let $z = x + iy$

Since it is given that $|z| \geq 2 \Rightarrow |z|^2 \geq 4 \Rightarrow x^2 + y^2 \geq 4$

$$\text{Now, } z + \frac{1}{z} = x + iy + \frac{1}{z} \Rightarrow \left| z + \frac{1}{z} \right|^2 = \left(x + \frac{1}{2} \right)^2 + y^2$$

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$$= x^2 + y^2 + \frac{1}{4} + x \geq 4 + \frac{1}{4} + x = \frac{17}{4} + x$$

Now minimum value of $\left|z + \frac{1}{2}\right|^2$ depends on x .

Since $|z| \geq 2$, so minimum value that x can take is -2 , when $y = 0$.

$$\therefore \left|z + \frac{1}{2}\right|^2 \geq \frac{17}{4} - 2 = \frac{9}{4} \Rightarrow \left|z + \frac{1}{2}\right| \geq \frac{3}{2}$$

49. (c) : Given $\cot \theta = \frac{-5}{12}$

Now, $\operatorname{cosec}^2 \theta = \cot^2 \theta + 1 = \left(\frac{-5}{12}\right)^2 + 1$

$$\Rightarrow \operatorname{cosec}^2 \theta = 1 + \frac{25}{144} = \frac{169}{144} = \left(\frac{13}{12}\right)^2$$

$$\Rightarrow \operatorname{cosec} \theta = \frac{13}{12} \left[\because \frac{\pi}{2} < \theta < \pi \right] \Rightarrow \sin \theta = \frac{12}{13}$$

50. (c) : Consider $\tan \frac{\pi}{4} = \frac{2 \tan \pi/8}{1 - \tan^2 \pi/8}$

$$\Rightarrow 1 = \frac{2x}{1-x^2} \left[\text{Putting } \tan \frac{\pi}{8} = x \right] \left[\because \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \right]$$

$$\Rightarrow 1 - x^2 = 2x \Rightarrow x^2 + 2x - 1 = 0$$

$$\therefore x = \frac{-2 \pm \sqrt{4+4}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$$

So, $\tan \frac{\pi}{8} = \sqrt{2} - 1 \left[\because \frac{\pi}{8} \text{ lies in } 1^{\text{st}} \text{ quadrant} \right]$

51. (b) : Let a be the first term and d be the common difference of given A.P.

Given $a_5 = \frac{1}{7}$ and $a_7 = \frac{1}{5}$

$$\Rightarrow a + 4d = \frac{1}{7} \quad \dots(i) \quad \text{and} \quad a + 6d = \frac{1}{5} \quad \dots(ii)$$

Solving (i) and (ii), we get $d = \frac{1}{35}$ and $a = \frac{1}{35}$

Sum of first 35 terms, $S_{35} = \frac{35}{2} \left[2 \times \frac{1}{35} + (35-1) \frac{1}{35} \right]$

$$= \frac{35}{2} \left[\frac{2}{35} + \frac{34}{35} \right] = 18$$

52. (a) : Given G.P. is $1, \frac{1}{2}, \frac{1}{4}, \dots$

Now, $S_n = 1 \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}} \right)$

$$\Rightarrow \frac{1023}{512} = \frac{1 - \left(\frac{1}{2}\right)^n}{\frac{1}{2}} \Rightarrow \frac{1023}{1024} = 1 - \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \left(\frac{1}{2}\right)^n = 1 - \frac{1023}{1024} = \frac{1}{1024} \Rightarrow \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{10} \Rightarrow n = 10$$

53. (c) : Let α, β be the roots of the quadratic equation. Given, A.M. of α and β is 8

$$\Rightarrow \frac{\alpha + \beta}{2} = 8 \Rightarrow \alpha + \beta = 16$$

Also, G.M. of α and β is 5 $\Rightarrow \sqrt{\alpha\beta} = 5 \Rightarrow \alpha\beta = 25$

$\therefore$ The required quadratic equation is $x^2 - 16x + 25 = 0$.

54. (a) : The given equation is

$$x^2 - (2a + b)x + (2a^2 + b^2 - b + \frac{1}{2}) = 0$$

Since roots are real $\therefore D \geq 0$

$$\Rightarrow (2a + b)^2 - 4(2a^2 + b^2 - b + \frac{1}{2}) \geq 0$$

$$\Rightarrow -4a^2 - 3b^2 + 4ab + 4b - 2 \geq 0$$

$$\Rightarrow 4a^2 + 3b^2 - 4ab - 4b + 2 \leq 0$$

$$\Rightarrow 4a^2 + b^2 - 4ab + 2(b^2 + 1 - 2b) \leq 0$$

$$\Rightarrow (2a - b)^2 + 2(b - 1)^2 \leq 0$$

$$\Rightarrow (2a - b)^2 = 0, (b - 1)^2 = 0 \Rightarrow 2a = b, b = 1$$

55. (e) : Given ${}^5P_r = {}^6P_{r-1}$

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{[6-(r-1)]!} \Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(7-r)!}$$

$$\Rightarrow \frac{(7-r)!}{(5-r)!} = \frac{6!}{5!} \Rightarrow (7-r)(6-r) = 6$$

$$\Rightarrow 42 - 13r + r^2 = 6 \Rightarrow r^2 - 13r + 36 = 0$$

$$\Rightarrow r^2 - 9r - 4r + 36 \Rightarrow r(r-9) - 4(r-9) = 0$$

$$\Rightarrow (r-4)(r-9) = 0$$

$$\Rightarrow r = 4 \text{ or } 9 \Rightarrow r = 4 \left[\because r \neq 9 \right]$$

56. (a) : Given ${}^nC_{2017} = {}^nC_{2016}$

$$\Rightarrow n = 2016 + 2017 \quad \left[\because nC_x = nC_y \Rightarrow n = x + y \right]$$

$$= 4033$$

$$\therefore {}^nC_{4033} = {}^{4033}C_{4033} = 1$$

57. (c) : Let $Q(h, k)$ be the image of the point $P(2, 1)$ in the line $2x - 3y + 1 = 0$.

Now, midpoint of $PQ = \left(\frac{2+h}{2}, \frac{1+k}{2} \right)$

Let R be any point on the line, so coordinates of R are

$$\left(\frac{2+h}{2}, \frac{1+k}{2} \right)$$





Sine R lies on the given line, so it will satisfy the equation of line.

$$\therefore 2\left(\frac{2+h}{2}\right) - 3\left(\frac{1+k}{2}\right) + 1 = 0$$

$$\Rightarrow 2(2+h) - 3(1+k) + 2 = 0$$

$$\Rightarrow 4 + 2h - 3 - 3k + 2 = 0$$

$$\Rightarrow 2h - 3k + 3 = 0$$

Also, PQ is perpendicular to given line.

$$\therefore \text{Slope of given line} \times \text{Slope of PQ} = -1$$

$$\Rightarrow \frac{2}{3} \cdot \frac{k-1}{h-2} = -1 \Rightarrow \frac{k-1}{h-2} = \frac{-3}{2}$$

$$\Rightarrow 2k - 2 = -3h + 6 \Rightarrow 3h + 2k - 8 = 0$$

$$\text{Solving (i) and (ii), we get } h = \frac{18}{13} \text{ and } k = \frac{25}{13}$$

$$\therefore \text{Image of point } P(2, 1) \text{ is } \left(\frac{18}{13}, \frac{25}{13}\right).$$

58. (a) : The given lines are

$$x^2 - 8x + 12 = 0$$

$$\text{and } y^2 - 14y + 45 = 0$$

$$\Rightarrow (x-6)(x-2) = 0$$

$$\text{and } (y-5)(y-9) = 0$$

Thus, the sides of square are

$$x = 2, x = 6, y = 5, y = 9$$

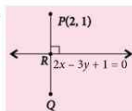
$$\therefore \text{Centre of the circle} = \left(\frac{2+6}{2}, \frac{5+9}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{14}{2}\right) = (4, 7) \therefore a + b = 4 + 7 = 11.$$

59. (c) : The given equation of parabola is

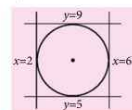
$$y^2 + 4y + 4x + 2 = 0 \Rightarrow (y+2)^2 + 4x - 2 = 0$$

$$\Rightarrow (y+2)^2 = -4x + 2 = -4\left(x - \frac{1}{2}\right) \Rightarrow Y^2 = -4X,$$



...(i)

...(ii)



where $Y = y + 2$ and $X = x - \frac{1}{2}$

Now, directrix of $Y^2 = -4X$ is $X = 1$.

$$\therefore \text{Directrix of given parabola is } \left(x - \frac{1}{2}\right) = 1 \Rightarrow x = \frac{3}{2}$$

60. (a) : The given equation of hyperbola is

$$\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$$

$$\text{Now, } e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{\sin^2 \alpha}{\cos^2 \alpha}} = \frac{1}{\cos \alpha}$$

$\therefore$ Foci of hyperbola = $(\pm ae, 0)$

$$= \left(\pm \cos \alpha \times \frac{1}{\cos \alpha}, 0\right) = (\pm 1, 0)$$

- | | | | | |
|------------|----------|-------------|----------|----------|
| 61. (a) | 62. (b) | 63. (d) | 64. (a) | 65. (b) |
| 66. (c) | 67. (a) | 68. (b) | 69. (b) | |
| 70. (None) | 71. (a) | 72. (d) | 73. (b) | |
| 74. (d) | 75. (c) | 76. (a) | 77. (a) | 78. (d) |
| 79. (d) | 80. (a) | 81. (b) | 82. (c) | 83. (a) |
| 84. (e) | 85. (e) | 86. (b) | 87. (b) | 88. (d) |
| 89. (c) | 90. (b) | 91. (c) | 92. (d) | 93. (a) |
| 94. (e) | 95. (c) | 96. (a) | 97. (b) | 98. (a) |
| 99. (b) | 100. (c) | 101. (b) | 102. (a) | 103. (c) |
| 104. (a) | 105. (d) | 106. (None) | 107. (c) | |
| 108. (e) | 109. (e) | 110. (e) | 111. (a) | 112. (c) |
| 113. (b) | 114. (a) | 115. (a) | 116. (c) | 117. (b) |
| 118. (e) | 119. (e) | 120. (b) | | |

For detailed solutions refer to
MTG Kerala PET Chapterwise Solutions

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CONCEPT MAP

TRIGONOMETRIC FUNCTIONS

Class XI

Relationship between Degree and Radian

- 1 radian = $\frac{180^\circ}{\pi} = 57^\circ 16'$ approximately
- $1^\circ = \frac{\pi}{180^\circ}$ radian = 0.01746 radian approximately

Sum and Difference of Two Angles

- $\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$
- $\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$
- $\tan(\theta \pm \phi) = \frac{\tan \theta \pm \tan \phi}{1 \mp \tan \theta \cdot \tan \phi}$
- $\cot(\theta \pm \phi) = \frac{\cot \theta \cdot \cot \phi \mp 1}{\cot \theta \pm \cot \phi}$
- $\sin \theta + \sin \phi = 2 \sin \left(\frac{\theta + \phi}{2} \right) \cos \left(\frac{\theta - \phi}{2} \right)$
- $\sin \theta - \sin \phi = 2 \cos \left(\frac{\theta + \phi}{2} \right) \sin \left(\frac{\theta - \phi}{2} \right)$
- $\cos \theta + \cos \phi = 2 \cos \left(\frac{\theta + \phi}{2} \right) \cos \left(\frac{\theta - \phi}{2} \right)$
- $\cos \theta - \cos \phi = -2 \sin \left(\frac{\theta + \phi}{2} \right) \sin \left(\frac{\theta - \phi}{2} \right)$
- $2 \sin \theta \cos \phi = \sin(\theta + \phi) + \sin(\theta - \phi)$
- $2 \cos \theta \sin \phi = \sin(\theta + \phi) - \sin(\theta - \phi)$
- $2 \cos \theta \cos \phi = \cos(\theta + \phi) + \cos(\theta - \phi)$
- $2 \sin \theta \sin \phi = \cos(\theta - \phi) - \cos(\theta + \phi)$

Trigonometric Equations

- $\sin \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- $\cos \theta = 0 \Rightarrow \theta = (2n+1)\pi/2, n \in \mathbb{Z}$
- $\tan \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{Z}$
- $\cot \theta = 0 \Rightarrow \theta = (2n+1)\pi/2, n \in \mathbb{Z}$
- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$
- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$
- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in \mathbb{Z}$

Functions

Domain

Range

$\sin \theta$	$\mathbb{R}$	$[-1, 1]$
$\cos \theta$	$\mathbb{R}$	$[-1, 1]$
$\tan \theta$	$\mathbb{R} - \left\{ (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \right\}$	$\mathbb{R}$
$\sec \theta$	$\mathbb{R} - \left\{ (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \right\}$	$\mathbb{R} - (-1, 1)$
$\operatorname{cosec} \theta$	$\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$	$\mathbb{R} - (-1, 1)$
$\cot \theta$	$\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$	$\mathbb{R}$

Multiple Angles

- $\sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1$
 $= 1 - 2 \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$
- $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$
- $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$
- $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$
- $\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

Properties of Triangles

- Sine rule:** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
- Cosine rule:**
 $a^2 = b^2 + c^2 - 2bc \cos A, b^2 = a^2 + c^2 - 2ac \cos B,$
 $c^2 = a^2 + b^2 - 2ab \cos C$
- Napier's rule:**
 $\tan \left(\frac{B-C}{2} \right) = \left(\frac{b-c}{b+c} \right) \cot \frac{A}{2},$
 $\tan \left(\frac{C-A}{2} \right) = \left(\frac{c-a}{c+a} \right) \cot \frac{B}{2},$
 $\tan \left(\frac{A-B}{2} \right) = \left(\frac{a-b}{a+b} \right) \cot \frac{C}{2}$

DETERMINANTS

CONCEPT MAP

Class XII

DETERMINANT

Corresponding to every square matrix A , there exists a number called the determinant of A and denoted by $|A|$.

$$\text{If } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \text{ then } |A| = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}).$$

Properties of Determinants

- The value of a determinant remains unaltered if its rows and columns are interchanged.
- If two rows (or columns) of a determinant are inter-changed, the value of the determinant is multiplied by -1 .
- If any two rows (or columns) of a determinant are identical, then the value of determinant is zero.
- If the elements of a row (or column) of a determinant are multiplied by any scalar, then the value of the new determinant is equal to same scalar times the value of the original determinant.
- If each element of any row (or column) of a determinant is the sum of two numbers, then the determinant is expressible as the sum of two determinants of the same order.

Minors and Cofactors

- For any matrix $A = [a_{ij}]_{n \times n}$, if we leave the row and the column of the element a_{ij} , then the value of determinant thus obtained is called the minor of a_{ij} and it is denoted by M_{ij} .

The minor M_{ij} multiplied by $(-1)^{i+j}$ is called the cofactor of the element a_{ij} and it is denoted by A_{ij} .

$$\therefore A_{ij} = (-1)^{i+j} M_{ij}$$

Adjoint of a Matrix

Adjoint of a matrix A , denoted by $\text{adj } A$, is defined as the transpose of the cofactors matrix of A .

Properties : If A is non-singular matrix of order n , then

- $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$
- $|A(\text{adj } A)| = |A|^n$
- $\text{adj}(AB) = (\text{adj } B) \cdot (\text{adj } A)$
- $|\text{adj } A| = |A|^{n-1}$
- $\text{adj}(\text{adj } A) = |A|^{n-2} A$
- $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

Area of a Triangle

- Let ABC be a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then area of ΔABC is

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- The area of the triangle formed by three collinear points is zero.

Singular and Non-Singular Matrices

A square matrix A of order n is said to be

- Singular** if $|A| = 0$
- Non-singular** if $|A| \neq 0$
- If A and B are non-singular matrices of the same order, then AB and BA are also non singular matrices of the same order.

Inverse of a Matrix

For any square matrix A , inverse of A is denoted by A^{-1} .

$$\text{as } A^{-1} = \frac{1}{|A|} (\text{adj } A), |A| \neq 0$$

Properties :

- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})^T$
- $(AB)^{-1} = B^{-1} A^{-1}$
- $(ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

Solution of System of Linear Equations

Let $AX = B$ be the given system of equations:

- If $|A| \neq 0$, the system is consistent and has unique solution.
- If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system is inconsistent and hence it has no solution.
- If $|A| = 0$ and $(\text{adj } A)B = O$, then the system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.



SOLVED PAPER 2019

Karnataka CET

1. The inverse of the matrix $\begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & -5 & 5 \\ -1 & -6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & -2 \end{bmatrix}$

2. If P and Q are symmetric matrices of the same order then $PQ - QP$ is
(a) identity matrix (b) zero matrix
(c) symmetric matrix (d) skew symmetric matrix

3. If $3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$ and $2B - 3A' =$

$\begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$ then $B =$

(a) $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -18 \\ 4 & -16 \\ -5 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & -4 \end{bmatrix}$

4. If $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then $|ABB'| =$
(a) 50 (b) 100
(c) -250 (d) 250

5. If the value of a third order determinant is 16, then the value of the determinant formed by replacing each of its elements by its cofactor is
(a) 96 (b) 256 (c) 48 (d) 16

6. $\int x^3 \sin 3x \, dx =$

(a) $-\frac{x^3 \cos 3x}{3} - \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$

(b) $-\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$

(c) $\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} - \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$

(d) $-\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} - \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$

7. The area of the region above X -axis included between the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2x$ in square units is

(a) $\frac{\pi}{4} - \frac{3}{2}$ (b) $\frac{2}{3} - \frac{\pi}{4}$ (c) $\frac{3}{2} - \frac{\pi}{4}$ (d) $\frac{\pi}{4} - \frac{2}{3}$

8. The area of the region bounded by Y -axis, $y = \cos x$ and $y = \sin x$; $0 \leq x \leq \frac{\pi}{2}$ is

(a) $(\sqrt{2} - 1)$ sq. units (b) $(\sqrt{2} + 1)$ sq. units

(c) $\sqrt{2}$ sq. units (d) $(2 - \sqrt{2})$ sq. units

9. The integrating factor of the differential equation $(2x + 3y^2) dy = y \, dx$ ($y > 0$) is

(a) e^y (b) $\frac{1}{x}$ (c) $-\frac{1}{y^2}$ (d) $\frac{1}{y^2}$

10. The equation of the curve passing through the point $(1, 1)$ such that the slope of the tangent at any point (x, y) is equal to the product of its coordinates is

(a) $2 \log x = y^2 - 1$ (b) $2 \log y = x^2 - 1$
(c) $2 \log y = x^2 + 1$ (d) $2 \log x = y^2 + 1$



11. Foot of the perpendicular drawn from the point (1, 3, 4) to the plane $2x - y + z = 0$ is

(a) $(-1, 4, 3)$ (b) $(1, 2, -3)$
(c) $(0, -4, -7)$ (d) $(-3, 5, 2)$

12. Acute angle between the line $\frac{x-5}{2} = \frac{y+1}{-1} = \frac{z+4}{1}$ and the plane $3x - 4y - z + 5 = 0$ is

(a) $\cos^{-1}\left(\frac{9}{\sqrt{364}}\right)$ (b) $\cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$
(c) $\sin^{-1}\left(\frac{9}{\sqrt{364}}\right)$ (d) $\sin^{-1}\left(\frac{5}{2\sqrt{13}}\right)$

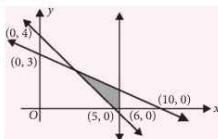
13. The distance of the point (1, 2, 1) from the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ is

(a) $\frac{2\sqrt{3}}{5}$ (b) $\frac{\sqrt{5}}{3}$ (c) $\frac{2\sqrt{5}}{3}$ (d) $\frac{20}{3}$

14. XY-plane divides the line joining the points A(2, 3, -5) and B(-1, -2, -3) in the ratio

(a) 2 : 1 internally (b) 5 : 3 internally
(c) 3 : 2 externally (d) 5 : 3 externally

15. The shaded region in the figure is the solution set of the inequations



(a) $4x + 5y \geq 20$, $3x + 10y \leq 30$, $x \leq 6$, $x, y \geq 0$
(b) $4x + 5y \leq 20$, $3x + 10y \leq 30$, $x \leq 6$, $x, y \geq 0$
(c) $4x + 5y \geq 20$, $3x + 10y \leq 30$, $x \geq 6$, $x, y \geq 0$
(d) $4x + 5y \leq 20$, $3x + 10y \leq 30$, $x \geq 6$, $x, y \geq 0$

16. The constant term in the expansion of

$$\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix} \text{ is}$$

(a) 0 (b) -10 (c) 2 (d) 6

17. If $[x]$ represents the greatest integer function and

$$f(x) = x - [x] - \cos x, \text{ then } f'\left(\frac{\pi}{2}\right) =$$

(a) 0 (b) 2
(c) 1 (d) does not exist

18. If $f(x) = \begin{cases} \frac{\sin 3x}{e^{2x}-1}; & x \neq 0 \\ k-2; & x=0 \end{cases}$ is continuous at $x=0$, then $k =$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{9}{5}$ (d) $\frac{2}{3}$

19. If $f(x) = \sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$, then $f'(0) =$

(a) $2 \log 2$ (b) $\frac{2 \log 2}{5}$ (c) $\log 2$ (d) $\frac{4 \log 2}{5}$

20. If $x = a \sec^2 \theta$, $y = a \tan^2 \theta$, then $\frac{d^2 y}{dx^2} =$

(a) $2a$ (b) 0 (c) 1 (d) 4

21. If α and β are roots of the equation $x^2 + x + 1 = 0$, then $\alpha^2 + \beta^2$ is

(a) 1 (b) $\frac{-1-i\sqrt{3}}{2}$
(c) $\frac{-1+i\sqrt{3}}{2}$ (d) -1

22. The number of 4 digit numbers without repetition that can be formed using the digits 1, 2, 3, 4, 5, 6, 7 in which each number has two odd digits and two even digits is

(a) 432 (b) 450 (c) 436 (d) 454

23. The number of terms in the expansion of $(x^2 + y^2)^{25} - (x^2 - y^2)^{25}$ after simplification is

(a) 0 (b) 26 (c) 13 (d) 50

24. The third term of a G.P. is 9. The product of its first five terms is

(a) 3^5 (b) 3^{10} (c) 3^9 (d) 3^{12}

25. A line cuts off equal intercepts on the co-ordinate axes. The angle made by this line with the positive direction of X-axis is

(a) 45° (b) 120° (c) 90° (d) 135°

26. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is

(a) $\frac{4}{5}$ (b) $\frac{3}{4}$ (c) $\frac{3}{5}$ (d) $\frac{9}{16}$

27. $\sum_{r=1}^n (2r-1) = x$ then $\lim_{n \rightarrow \infty} \left[\frac{1^3}{x^2} + \frac{2^3}{x^2} + \frac{3^3}{x^2} + \dots + \frac{n^3}{x^2} \right] =$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) 4

28. The negation of the statement "All continuous functions are differentiable" is

(a) All continuous functions are not differentiable
(b) Some continuous functions are not differentiable
(c) Some continuous functions are differentiable
(d) All differentiable functions are continuous



29. Mean and standard deviation of 100 items are 50 and 4 respectively. The sum of all squares of the items is
(a) 251600 (b) 266000
(c) 256100 (d) 261600
30. Two letters are chosen from the letters of the word 'EQUATIONS'. The probability that one is vowel and the other is consonant is
(a) $\frac{8}{9}$ (b) $\frac{3}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$
31. $f: R \rightarrow R$ and $g: [0, \infty) \rightarrow R$ is defined by $f(x) = x^2$ and $g(x) = \sqrt{x}$. Which one of the following is not true?
(a) $gof(4) = 4$ (b) $fog(2) = 2$
(c) $fog(-4) = 4$ (d) $gof(-2) = 2$
32. If $A = \{x|x \in N, x \leq 5\}$, $B = \{x|x \in Z, x^2 - 5x + 6 = 0\}$, then the number of onto functions from A to B is
(a) 2 (b) 30 (c) 23 (d) 32
33. On the set of positive rationals, a binary operation $*$ is defined by $a * b = \frac{2ab}{5}$.
If $2 * x = 3^{-1}$, then $x =$
(a) $\frac{1}{6}$ (b) $\frac{2}{5}$ (c) $\frac{5}{12}$ (d) $\frac{125}{48}$
34. $\cos \left[2 \sin^{-1} \frac{3}{4} + \cos^{-1} \frac{3}{4} \right] =$
(a) $\frac{-3}{4}$ (b) $\frac{3}{5}$
(c) $\frac{3}{4}$ (d) does not exist
35. If $a + \frac{\pi}{2} < 2 \tan^{-1} x + 3 \cot^{-1} x < b$ then ' a ' and ' b ' are respectively
(a) 0 and π (b) 0 and 2π
(c) $\frac{\pi}{2}$ and 2π (d) $\frac{-\pi}{2}$ and $\frac{\pi}{2}$
36. If $|3x - 5| \leq 2$, then
(a) $-1 \leq x \leq \frac{7}{3}$ (b) $1 \leq x \leq \frac{9}{3}$
(c) $1 \leq x \leq \frac{7}{3}$ (d) $-1 \leq x \leq \frac{9}{3}$
37. A random variable ' X ' has the following probability distribution:
- | | | | | | | | |
|--------|-------|------|-----|------|--------|-------|-----------|
| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $P(X)$ | $k-1$ | $3k$ | k | $3k$ | $3k^2$ | k^2 | $k^2 + k$ |
- Then the value of k is
(a) $\frac{1}{5}$ (b) $\frac{2}{7}$ (c) -2 (d) $\frac{1}{10}$
38. If A and B are two events of a sample space S such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.5$, then $P(A|B) =$
(a) $\frac{3}{10}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$
39. If ' X ' has a binomial distribution with parameters $n = 6$, p and $P(X = 2) = 12$, $P(X = 3) = 5$, then $p =$
(a) $\frac{5}{12}$ (b) $\frac{1}{2}$ (c) $\frac{16}{21}$ (d) $\frac{5}{16}$
40. A man speaks truth 2 out of 3 times. He picks one of the natural numbers in the set $S = \{1, 2, 3, 4, 5, 6, 7\}$ and reports that it is even. The probability that it is actually even is
(a) $\frac{2}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$
41. The order of the differential equation $y = C_1 e^{C_2 + x} + C_3 e^{C_4 + x}$ is
(a) 1 (b) 3 (c) 2 (d) 4
42. If $|\vec{a}| = 16$, $|\vec{b}| = 4$ then, $\sqrt{|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2} =$
(a) 4 (b) 16 (c) 8 (d) 64
43. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and the projection of $\vec{a}$ in the direction of $\vec{b}$ is -2, then $|\vec{a}| =$
(a) 4 (b) 2 (c) 3 (d) 1
44. A unit vector perpendicular to the plane containing the vectors $\hat{i} + 2\hat{j} + \hat{k}$ and $-2\hat{i} + \hat{j} + 3\hat{k}$ is
(a) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (b) $\frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$
(c) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (d) $\frac{-\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$
45. $[\vec{a} + 2\vec{b} - \vec{c}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}] =$
(a) 0 (b) $2[\vec{a}, \vec{b}, \vec{c}]$
(c) $[\vec{a}, \vec{b}, \vec{c}]$ (d) $3[\vec{a}, \vec{b}, \vec{c}]$
46. If $\sqrt[3]{y}\sqrt{x} = \sqrt[6]{(x+y)^5}$, then $\frac{dy}{dx} =$
(a) $\frac{x}{y}$ (b) $x - y$ (c) $x + y$ (d) $\frac{y}{x}$
47. Rolle's theorem is not applicable in which one of the following cases?
(a) $f(x) = x^2 - 4x + 5$ in $[1, 3]$
(b) $f(x) = |x|$ in $[-2, 2]$
(c) $f(x) = x^2 - x$ in $[0, 1]$
(d) $f(x) = [x]$ in $[2.5, 2.7]$





48. The interval in which the function $f(x) = x^3 - 6x^2 + 9x + 10$ is increasing in
 (a) $(-\infty, 1) \cup (3, \infty)$ (b) $[1, 3]$
 (c) $(-\infty, 1] \cup [3, \infty)$ (d) $(-\infty, -1] \cup [3, \infty)$
49. The sides of an equilateral triangle are increasing at the rate of 4 cm/sec. The rate at which its area is increasing, when the side is 14 cm
 (a) $10\sqrt{3}$ cm²/sec (b) 42 cm²/sec
 (c) $14\sqrt{3}$ cm²/sec (d) 14 cm²/sec
50. The value of $\sqrt{24.99}$ is
 (a) 4.999 (b) 5.001 (c) 4.899 (d) 4.897
51. $\int_{-3}^3 \cot^{-1} x \, dx =$
 (a) 3π (b) 6π (c) 0 (d) 3
52. $\int \frac{1}{\sqrt{x} + x\sqrt{x}} dx =$
 (a) $2 \log(\sqrt{x} + 1) + C$ (b) $\tan^{-1} \sqrt{x} + C$
 (c) $\frac{1}{2} \tan^{-1} \sqrt{x} + C$ (d) $2 \tan^{-1} \sqrt{x} + C$
53. $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx = A \log|x-1| + B \log|x+2| + C \log|x-3| + K$, then A, B, C are respectively
 (a) $\frac{-1}{6}, \frac{1}{3}, \frac{-1}{2}$ (b) $\frac{1}{6}, \frac{-1}{3}, \frac{1}{3}$
 (c) $\frac{1}{6}, \frac{1}{3}, \frac{1}{5}$ (d) $\frac{-1}{6}, \frac{-1}{3}, \frac{1}{2}$
54. $\int_0^2 [x^2] dx =$
 (a) $5 - \sqrt{2} - \sqrt{3}$ (b) $5 - \sqrt{2} + \sqrt{3}$
 (c) $5 + \sqrt{2} - \sqrt{3}$ (d) $-5 - \sqrt{2} - \sqrt{3}$
55. $\int_0^1 \sqrt{\frac{1+x}{1-x}} dx =$
 (a) $\frac{\pi}{2} - 1$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{2} + 1$ (d) $\frac{1}{2}$
56. If U is the universal set with 100 elements; A and B are two sets such that $n(A) = 50$, $n(B) = 60$, $n(A \cap B) = 20$ then $n(A' \cap B') =$
 (a) 40 (b) 90 (c) 20 (d) 10
57. The domain of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{x^2 - 7x + 12}$ is
 (a) $(-\infty, 3] \cup [4, \infty)$ (b) $(-\infty, 3] \cap [4, \infty)$
 (c) $(-\infty, 3] \cup (4, \infty)$ (d) $(3, 4)$
58. If $\cos x = |\sin x|$, then the general solution is
 (a) $x = n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$
 (b) $x = n\pi + (-1)^n \frac{\pi}{4}$, $n \in \mathbb{Z}$
 (c) $x = 2n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$
 (d) $x = (2n+1)\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$
59. $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ =$
 (a) 2 (b) 4 (c) 3 (d) 1
60. If $P(n): 2^n < n!$ then the smallest positive integer for which $P(n)$ is true, is
 (a) 2 (b) 4 (c) 3 (d) 5

SOLUTIONS

1. (b): Let $A = \begin{bmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$

$\therefore |A| = 2(3-0) - 1(5-0) = 6-5=1 (\neq 0)$

So A^{-1} exists.

Now, $\operatorname{adj} A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{\operatorname{adj} A}{|A|} = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}$

2. (d): We have, $P = P^T$ and $Q = Q^T$... (i)

Now, $(PQ - QP)^T = (PQ)^T - (QP)^T$
 $= Q^T P^T - P^T Q^T = QP - PQ$ [Using (i)]
 $= -(PQ - QP)$

Hence, $(PQ - QP)$ is skew symmetric matrix.

3. (a): We have, $3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$... (i)

and $2B - 3A' = \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$

Now, $(2B - 3A')' = (2B)' - (3A')' = 2B' - 3A$

$\therefore 2B' - 3A = \begin{bmatrix} -1 & 4 & -5 \\ 18 & 0 & -7 \end{bmatrix}$... (ii)

Adding (i) and (ii), we get

$$6B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix} + \begin{bmatrix} -1 & 4 & -5 \\ 18 & 0 & -7 \end{bmatrix}$$

$$\Rightarrow 6B' = \begin{bmatrix} 6 & -6 & 12 \\ 18 & 6 & 24 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$$

4. (c): We have, $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ and

$$B' = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\therefore |A| = 2 - 12 = -10, |B| = 4 + 1 = 5 \text{ and } |B'| = 4 + 1 = 5$$

$$\text{Now, } |ABB'| = |A| |B| |B'| = (-10) \times (5) \times (5) \quad [\text{Using (i)}]$$

$$= -250$$

5. (b): Let A be the matrix of order 3.

Given, $|A| = 16$

The determinant formed by replacing each elements of the matrix A by its cofactor is transpose of $\text{adj } A$.

$$\text{Now, } |(\text{adj } A)^T| = |\text{adj } A| = |A|^{3-1}$$

$$= |A|^2 = (16)^2 = 256$$

6. (b): Let $I = \int x^3 \sin 3x \, dx$

$$= \int x^3 \sin 3x \, dx - \int \left[\frac{d}{dx} (x^3) \right] \sin 3x \, dx$$

$$= x^3 \left[\frac{-\cos 3x}{3} \right] + \frac{1}{3} \int 3x^2 \cos 3x \, dx$$

$$= -\frac{x^3}{3} \cos 3x + \frac{1}{3} \left[3x^2 \int \cos 3x \, dx \right. \\ \left. - \int \left[\frac{d}{dx} (3x^2) \right] \int \cos 3x \, dx \right] dx$$

$$= -\frac{x^3 \cos 3x}{3} + \left[x^2 \left(\frac{\sin 3x}{3} \right) - \frac{1}{3} \int 2x \sin 3x \, dx \right]$$

$$= -\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} - \frac{2}{3} \left[x \left(\frac{-\cos 3x}{3} \right) + \frac{1}{3} \int \cos 3x \, dx \right]$$

$$= -\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2}{9} \left(\frac{\sin 3x}{3} \right) + C$$

$$= -\frac{x^3 \cos 3x}{3} + \frac{x^2 \sin 3x}{3} + \frac{2x \cos 3x}{9} - \frac{2 \sin 3x}{27} + C$$

7. (d): Given equation of circle is $x^2 + y^2 = 2x$
 $\Rightarrow (x-1)^2 + (y-0)^2 = 1$

So, the circle has centre at $(1, 0)$ and radius 1 unit.

Now, both curves intersect at $(1, 1)$ and $(1, -1)$.

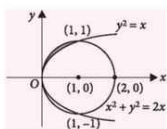
$\therefore$ Required shaded area

$$= \int_0^1 (\sqrt{1-(x-1)^2} - \sqrt{x}) \, dx$$

$$= \left[\frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) - \frac{2}{3} x^{3/2} \right]_0^1$$

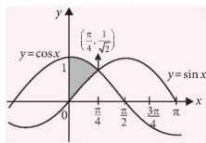
$$= \left(0 + 0 - \frac{2}{3} \right) - \left(0 + \frac{1}{2} \sin^{-1}(-1) - 0 \right)$$

$$= -\frac{2}{3} - \frac{1}{2} \left(-\frac{\pi}{2} \right) = \left(\frac{\pi}{4} - \frac{2}{3} \right) \text{ sq. units}$$



8. (a): We have, $y = \cos x$ and $y = \sin x$

Both curves intersect at $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}} \right)$.



$\therefore$ Required shaded area = $\int_0^{\pi/4} (\cos x - \sin x) \, dx$

$$= [\sin x + \cos x]_0^{\pi/4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1$$

$$= \frac{2}{\sqrt{2}} - 1 = (\sqrt{2} - 1) \text{ sq. units}$$

9. (d): The given differential equation is

$$(2x + 3y^2) \, dy = y \, dx \Rightarrow \frac{2x + 3y^2}{y} = \frac{dx}{dy}$$

$$\Rightarrow \frac{2x}{y} + 3y = \frac{dx}{dy} \Rightarrow \frac{dx}{dy} - \frac{2}{y} x = 3y$$

$$\therefore \text{I.F.} = e^{-\int \frac{2}{y} \, dy} = e^{-2 \log y} = y^{-2} = \frac{1}{y^2}$$

10. (b): We have, $\frac{dy}{dx} = xy \Rightarrow \frac{dy}{y} = x \, dx$

Integrating both sides, we get

$$\log y = \frac{x^2}{2} + C$$

Since (i) passes through the point $(1, 1)$.

... (i)



$$\therefore \log(1) = \frac{1}{2} + C \Rightarrow C = -\frac{1}{2}$$

$$\text{From (i), } \log y = \frac{x^2}{2} - \frac{1}{2} \Rightarrow 2 \log y = x^2 - 1$$

11. (a): Let $P(x_1, y_1, z_1)$ be the foot of the perpendicular drawn from the point $A(1, 3, 4)$ to the plane $2x - y + z + 3 = 0$.

The d.r.'s of PA are $x_1 - 1, y_1 - 3, z_1 - 4$.

And the d.r.'s of normal to the given plane are $2, -1, 1$.

$\therefore$ Equation of line AP is given by

$$\frac{x_1 - 1}{2} = \frac{y_1 - 3}{-1} = \frac{z_1 - 4}{1} = \lambda \quad (\text{say})$$

$$\Rightarrow x_1 = 2\lambda + 1, y_1 = 3 - \lambda \text{ and } z_1 = \lambda + 4$$

Since $P(x_1, y_1, z_1)$ lies on the plane $2x - y + z + 3 = 0$.

$$\therefore 2(2\lambda + 1) - 1(3 - \lambda) + \lambda + 4 + 3 = 0$$

$$\Rightarrow 4\lambda + 2 - 3 + \lambda + \lambda + 7 = 0 \Rightarrow 6\lambda = -6 \Rightarrow \lambda = -1$$

$\therefore$ Coordinates of P are $(-1, 4, 3)$.

12. (b): D.R.'s of the given line are $2, -1, 1$ and d.r.'s of normal to the given plane are $3, -4, -1$.

Let θ be the angle between the given line and plane.

$$\therefore \sin \theta = \frac{\left| \frac{6+4-1}{\sqrt{156}} \right|}{\frac{9}{\sqrt{6+4-1}}} = \frac{9}{\sqrt{6}\sqrt{26}} = \frac{9}{\sqrt{156}}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{9}{\sqrt{156}}\right) = \cos^{-1}\left(\frac{\sqrt{75}}{\sqrt{156}}\right) = \cos^{-1}\left(\frac{5}{2\sqrt{13}}\right)$$

13. (c): Given equation of line is

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2} = r \quad (\text{say})$$

$\therefore$ Coordinates of any point Q on the line are $(2r+1, r+2, 2r+3)$.

Let P be the point with coordinates $(1, 2, 1)$.

Now, d.r.'s of PQ are $2r+1-1, r+2-2, 2r+3-1$ or $2r, r, 2r+2$.

Also, d.r.'s of the given line are $2, 1, 2$.

Since PQ is perpendicular to the given line.

$$\therefore 4r + r + 4r + 4 = 0$$

$$\Rightarrow 9r = -4 \Rightarrow r = -\frac{4}{9}$$

So, coordinates of Q are $\left(\frac{1}{9}, \frac{14}{9}, \frac{19}{9}\right)$.

$$\therefore PQ = \sqrt{\left(1 - \frac{1}{9}\right)^2 + \left(2 - \frac{14}{9}\right)^2 + \left(1 - \frac{19}{9}\right)^2}$$

$$= \sqrt{\frac{64}{81} + \frac{16}{81} + \frac{100}{81}} = \frac{\sqrt{180}}{9} = \frac{2\sqrt{5}}{3}$$

14. (d): Let XY -plane divides the line joining the points $A(2, 3, -5)$ and $B(-1, -2, -3)$ in the ratio $k : 1$. Let the coordinates of point of intersection are (x_1, y_1, z_1) .

$$\therefore x_1 = \frac{-k+2}{k+1}, y_1 = \frac{-2k+3}{k+1}, z_1 = \frac{-3k-5}{k+1}$$

Since the plane is XY -plane, so, $z = 0$.

$$\therefore -3k - 5 = 0 \Rightarrow k = -\frac{5}{3}$$

So, XY -plane divides the line AB in the ratio $5 : 3$ externally.

15. (a): The equations of lines given in the figure are

$$\frac{x}{5} + \frac{y}{4} = 1 \Rightarrow 4x + 5y = 20;$$

$$\frac{x}{10} + \frac{y}{3} = 1 \Rightarrow 3x + 10y = 30; x = 6 \text{ and } y = 0$$

So inequations satisfying the shaded region are

$$4x + 5y \geq 20, 3x + 10y \leq 30, x \leq 6, x, y \geq 0$$

16. (d): Given,
$$\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$$

To find the constant term, putting $x = 0$, we get

$$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix} = 1(-2-1) + 1(1+2) + 2(-1+4) = -3 + 3 + 6 = 6$$

So, the constant term is 6.

17. (b): Given, $f(x) = x - [x] - \cos x$

$$\therefore f'(x) = 1 + \sin x \quad [\because [x] \text{ is an integer}]$$

$$\therefore f'\left(\frac{\pi}{2}\right) = 1 + \sin \frac{\pi}{2} = 1 + 1 = 2$$

18. (None of the options is correct):

Since $f(x)$ is continuous at $x = 0$.

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0) \Rightarrow \lim_{x \rightarrow 0} \frac{\sin 3x}{e^{2x} - 1} = k - 2 \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{3 \cos 3x}{2e^{2x}} = k - 2 \quad (\text{Using L' Hospital's rule})$$

$$\Rightarrow \frac{3}{2} = k - 2 \Rightarrow k = \frac{3}{2} + 2 = \frac{7}{2}$$

19. (c): Given, $f(x) = \sin^{-1} \left[\frac{2^{x+1}}{1+4^x} \right] = \sin^{-1} \left[\frac{2 \cdot 2^x}{1+2^{2x}} \right]$

$$= \sin^{-1} \left[\frac{2 \cdot 2^x}{1+(2^x)^2} \right]$$



Put $2^x = \tan \theta$

$$\therefore f(x) = \sin^{-1} \left[\frac{2 \tan \theta}{1 + \tan^2 \theta} \right] = \sin^{-1} [\sin 2\theta] = 2\theta$$

$$\therefore f(x) = 2 \tan^{-1}(2^x)$$

$$\therefore f'(x) = 2 \cdot \frac{1}{1 + (2^x)^2} \cdot 2^x \log 2$$

$$\text{Now, } f'(0) = \frac{2}{2} \cdot 1 \cdot \log 2 = \log 2$$

20. (b): We have, $x = a \sec^2 \theta$ and $y = a \tan^2 \theta$

$$\therefore \frac{dx}{d\theta} = 2a \sec \theta \cdot \sec \theta \tan \theta = 2a \sec^2 \theta \tan \theta$$

$$\frac{dy}{d\theta} = 2a \tan \theta \sec^2 \theta$$

$$\text{So, } \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2a \tan \theta \sec^2 \theta}{2a \sec^2 \theta \tan \theta} = 1 \quad \therefore \frac{d^2 y}{dx^2} = 0$$

21. (d): We have, $x^2 + x + 1 = 0$... (i)

Since α, β are roots of equation (i).

$$\therefore \alpha + \beta = -1, \alpha\beta = 1$$

$$\text{Now, } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2 = -1$$

22. (a): Given digits are 1, 2, 3, 4, 5, 6 and 7.

So, number of even digits are 3 and number of odd digits are 4.

Two even digits can be selected in ${}^3C_2$ ways and two odd digits can be selected in ${}^4C_2$ ways.

These 4 digits can be arranged in $4!$ ways.

So, total number of 4 digit numbers

$$= {}^3C_2 \times {}^4C_2 \times 4! = 3 \times 6 \times 24 = 432$$

23. (c): We know that, when n is odd, the number of terms in $(x+a)^n - (x-a)^n$ is $\frac{n+1}{2}$.

Given expansion is $(x^2 + y^2)^{25} - (x^2 - y^2)^{25}$.

So, number of terms in the given expansion is

$$\frac{25+1}{2} = 13.$$

24. (b): Let the first five terms of the G.P. are $\frac{a}{r^2}, \frac{a}{r}, a, ar$ and ar^2 .

Given, $a = 9$

So, product of first five terms $= a^5 = (9)^5 = (3^2)^5 = 3^{10}$

25. (d): Let the equation of line which cuts equal intercepts on the co-ordinate axes is $x + y = c$... (i)

$$\therefore 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1$$

Let θ be the angle made by (i) with positive direction of X -axis.

$$\therefore \tan \theta = -1 = -\tan \frac{\pi}{4} = \tan \left(\pi - \frac{\pi}{4} \right)$$

$$\Rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} = 135^\circ$$

26. (a): Given equation of ellipse is $9x^2 + 25y^2 = 225$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\text{So, eccentricity } (e) = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

27. (c): Given, $x = \sum_{r=1}^n (2r-1)$

$$\Rightarrow x = 1 + 3 + 5 + \dots + (2n-1) \Rightarrow x = n^2 \Rightarrow x^2 = n^4$$

$$\text{Now, } \lim_{n \rightarrow \infty} \left[\frac{1^3}{x^2} + \frac{2^3}{x^2} + \frac{3^3}{x^2} + \dots + \frac{n^3}{x^2} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{x^2}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2(n+1)^2}{4n^4} = \lim_{n \rightarrow \infty} \frac{n^4 \left(1 + \frac{1}{n}\right)^2}{4n^4} = \frac{1}{4}$$

28. (b)

29. (a): Given, mean $(\bar{x}) = 50$

Standard deviation $(\sigma) = 4$

$$\Rightarrow \sqrt{\frac{\sum x_i^2}{100} - (\bar{x})^2} = 4 \Rightarrow \frac{\sum x_i^2}{100} - (50)^2 = (4)^2$$

$$\Rightarrow \frac{\sum x_i^2}{100} = 16 + 2500 = 2516 \Rightarrow \sum x_i^2 = 251600$$

30. (d): The word 'EQUATIONS' has 9 letters, among them 5 are vowels and 4 are consonants.

$$\therefore \text{Required probability} = \frac{{}^5C_1 \times {}^4C_1}{{}^9C_2} = \frac{5 \times 4}{9 \times 4} = \frac{5}{9}$$

31. (c): Here $f: R \rightarrow R$ and $g: [0, \infty) \rightarrow R$ is defined by

$$f(x) = x^2 \text{ and } g(x) = \sqrt{x}$$

$$\text{Now, } g \circ f(4) = g(f(4)) = g(16) = \sqrt{16} = 4$$

$$f \circ g(2) = f(g(2)) = f(\sqrt{2}) = (\sqrt{2})^2 = 2$$

$$f \circ g(-4) = f(g(-4)), \text{ which is not defined.}$$

$$g \circ f(-2) = g(f(-2)) = g(4) = \sqrt{4} = 2$$

32. (b): We have $A = \{x | x \in N, x \leq 5\}$ and

$$B = \{x | x \in Z, x^2 - 5x + 6 = 0\}$$

$$\therefore A = \{1, 2, 3, 4, 5\} \text{ and } B = \{x | x \in Z, (x-3)(x-2) = 0\}$$

$$\Rightarrow B = \{2, 3\}$$

Now number of onto functions from a set containing 5 elements to a set containing 2 elements is given by $2^5 - 2 = 32 - 2 = 30$.





33. (d): Here * is defined by $a * b = \frac{2ab}{5}$

Now we know that $a * e = a \Rightarrow \frac{2ae}{5} = a \Rightarrow e = \frac{5}{2}$

Also, $a * a^{-1} = e \Rightarrow \frac{2aa^{-1}}{5} = \frac{5}{2} \Rightarrow a^{-1} = \frac{25}{4a}$

Now, $2 * x = 3^{-1} \Rightarrow \frac{2(2x)}{5} = \frac{25}{4(3)} \quad \left[\because a^{-1} = \frac{25}{4a} \right]$

$\Rightarrow x = \frac{125}{48}$

34. (a): We have $\cos \left[2\sin^{-1} \frac{3}{4} + \cos^{-1} \frac{3}{4} \right]$

$= \cos \left[2\sin^{-1} \frac{3}{4} + \frac{\pi}{2} - \sin^{-1} \frac{3}{4} \right]$

$= \cos \left[\frac{\pi}{2} + \sin^{-1} \frac{3}{4} \right] = -\sin \left(\sin^{-1} \frac{3}{4} \right) = -\frac{3}{4}$

35. (c): Here, $a + \frac{\pi}{2} < 2 \tan^{-1} x + 3 \cot^{-1} x < b$

$\Rightarrow a + \frac{\pi}{2} < 2(\tan^{-1} x + \cot^{-1} x) + \cot^{-1} x < b$

$\Rightarrow a + \frac{\pi}{2} < \pi + \cot^{-1} x < b \Rightarrow a - \frac{\pi}{2} < \cot^{-1} x < b - \pi$

$\Rightarrow a - \frac{\pi}{2} = 0$ and $b - \pi = \pi \quad [\because 0 < \cot^{-1} x < \pi]$

$\Rightarrow a = \frac{\pi}{2}$ and $b = 2\pi$

36. (c): Given, $|3x - 5| \leq 2 \Rightarrow -2 \leq 3x - 5 \leq 2$

$\Rightarrow 3 \leq 3x \leq 7 \Rightarrow 1 \leq x \leq \frac{7}{3}$

37. (a): Since $\Sigma P_i = 1$

$\therefore k - 1 + 3k + k + 3k + 3k^2 + k^2 + k^2 + k = 1$

$\Rightarrow 5k^2 + 9k - 2 = 0 \Rightarrow (5k - 1)(k + 2) = 0 \Rightarrow k = \frac{1}{5}$ or -2

Since $k = -2$ is not possible

So, $k = \frac{1}{5}$

38. (b): Given, $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.5$

$\Rightarrow \frac{P(A \cap B)}{P(B)} = 0.5 \Rightarrow P(A \cap B) = 0.5 P(B) = (0.5)(0.6) = 0.3$

Now, $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{0.6 - 0.3}{0.6} = \frac{0.3}{0.6} = \frac{1}{2}$

39. (None of the options is correct): Given, $P(X=2) = 12$

$\Rightarrow {}^6C_2 p^2 q^4 = 12 \quad \dots(i) \quad [\because n = 6 \text{ (given)}]$

Also, $P(X=3) = 5 \Rightarrow {}^6C_3 p^3 q^3 = 5 \quad \dots(ii)$

From (i) and (ii), $\frac{{}^6C_2 p^2 q^4}{{}^6C_3 p^3 q^3} = \frac{12}{5} \Rightarrow \frac{15q}{20p} = \frac{12}{5}$

$\Rightarrow 75q = 240p \Rightarrow 75(1-p) = 240p \quad [\because p+q=1]$

$\Rightarrow 75 - 75p = 240p \Rightarrow 315p = 75 \Rightarrow p = \frac{75}{315} = \frac{5}{21}$

40. (d): Given $S = \{1, 2, 3, 4, 5, 6, 7\}$.

Let E_1 and E_2 respectively be the events that an even number is picked and an odd number is picked.

$\therefore P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{4}{7}$

Let E be the event that man reports it is an even number.

$\therefore P(E|E_1) = \frac{2}{3}$ and $P(E|E_2) = \frac{1}{3}$.

Required probability $= P(E_1|E)$

$= \frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1) + P(E|E_2)P(E_2)}$

$= \frac{\frac{2}{3} \times \frac{3}{7}}{\frac{2}{3} \times \frac{3}{7} + \frac{1}{3} \times \frac{4}{7}} = \frac{6}{6+4} = \frac{6}{10} = \frac{3}{5}$

41. (a): Given $y = C_1 e^{C_2 x} + C_3 e^{C_4 x}$

$\Rightarrow y = C_1 e^{C_2 x} + C_3 e^{C_4 x} \Rightarrow y = (C_1 e^{C_2 x} + C_3 e^{C_4 x}) e^x$

$\Rightarrow y = A e^x$, where $A = C_1 e^{C_2 x} + C_3 e^{C_4 x} \quad \dots(i)$

Differentiating (i), w.r.t. x , we get

$\frac{dy}{dx} = A e^x = y \Rightarrow \frac{dy}{dx} = y$

$\therefore$ Order of the given differential equation $= 1$

42. (d): Consider $\sqrt{|\vec{a} \times \vec{b}|^2} + |\vec{a} \cdot \vec{b}|^2$

$= \sqrt{|\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta} + |\vec{a}| |\vec{b}| \cos^2 \theta$

$= \sqrt{|\vec{a}|^2 |\vec{b}|^2 (\sin^2 \theta + \cos^2 \theta)} = |\vec{a}| |\vec{b}| = 16 \times 4 = 64$

43. (a): Given angle between $\vec{a}$ and $\vec{b}$ i.e., $\theta = \frac{2\pi}{3}$

Now, projection of $\vec{a}$ along $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

$\Rightarrow -2 = \frac{|\vec{a}| |\vec{b}| \cos \theta}{|\vec{b}|} \Rightarrow -2 = |\vec{a}| \cos \frac{2\pi}{3}$

$\Rightarrow -2 = -|\vec{a}| \cos \frac{\pi}{3} \Rightarrow 2 = |\vec{a}| \frac{1}{2} \Rightarrow |\vec{a}| = 4$

44. (b): Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} + 3\hat{k}$

Now $\hat{n}$ be the unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$.

$\therefore \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$$\text{Now, } \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 1 & 3 \end{vmatrix}$$

$$= \hat{i}(6-1) - \hat{j}(3+2) + \hat{k}(1+4) = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{25+25+25} = 5\sqrt{3}$$

$$\therefore \hat{n} = \frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}} = -\left(\frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}\right)$$

$$\text{So, } \hat{n} \text{ can be } \frac{-\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}} \text{ also.}$$

$$45. (d): \text{ Consider } [\vec{a} + 2\vec{b} - \vec{c}, \vec{a} - \vec{b}, \vec{a} - \vec{b} - \vec{c}]$$

$$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot \{(\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})\}$$

$$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot \{-\vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{b} \times \vec{a} + \vec{b} \times \vec{c}\}$$

$$= (\vec{a} + 2\vec{b} - \vec{c}) \cdot \{-\vec{a} \times \vec{c} + \vec{b} \times \vec{c}\} = \vec{a} \cdot (\vec{b} \times \vec{c}) - 2\vec{b} \cdot (\vec{a} \times \vec{c})$$

$$= [\vec{a}, \vec{b}, \vec{c}] - 2[\vec{b}, \vec{a}, \vec{c}] = [\vec{a}, \vec{b}, \vec{c}] + 2[\vec{a}, \vec{b}, \vec{c}] = 3[\vec{a}, \vec{b}, \vec{c}]$$

$$46. (d): \sqrt[3]{y} \sqrt{x} = \sqrt[6]{(x+y)^5}$$

$$\Rightarrow (y^{1/3} x^{1/2})^{6/5} = (x+y)^{5/6 \times 6/5}$$

$$\Rightarrow y^{2/5} x^{3/5} = x+y$$

Differentiating w.r.t. x , we get

$$y^{2/5} \cdot \frac{3}{5} x^{-2/5} + x^{3/5} \cdot \frac{2}{5} y^{-3/5} \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left[1 - \frac{2}{5} \frac{x^{3/5}}{y^{3/5}} \right] = \frac{3}{5} \left(\frac{y}{x} \right)^{2/5} - 1$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{3y^{2/5} - 5x^{2/5}}{5y^{3/5} - 2x^{3/5}} \right] \frac{5y^{3/5}}{5x^{3/5} - 2x^{3/5}}$$

$$= \frac{y^{2/5} x^{3/5}}{x^{3/5} y^{2/5}} \cdot \frac{3y^{2/5} - 5x^{2/5}}{5y^{3/5} - 2x^{3/5}} \cdot \frac{y^{3/5}}{x^{2/5}}$$

$$= \left(\frac{3y^{2/5} x^{3/5} - 5x}{5y - 2x^{3/5} y^{2/5}} \right) \frac{y^{2/5+3/5}}{x^{2/5+3/5}} = \left(\frac{3(x+y) - 5x}{5y - 2(x+y)} \right) \frac{y}{x}$$

[Using (i)]

$$= \left(\frac{3y - 2x}{3y - 2x} \right) \frac{y}{x} = \frac{y}{x}$$

$$47. (b): (a) f(x) = x^2 - 4x + 5 \text{ in } [1, 3]$$

$$f(1) = f(3) = 2; f'(x) = 2x - 4 = 2(x - 2)$$

$$\text{Put } f'(x) = 0 \Rightarrow x = 2 \text{ or } c = 2 \in [1, 3]$$

$\therefore$ Rolle's theorem is applicable.

$$(b) f(x) = |x| \text{ in } [-2, 2]$$

Since $f(x) = |x|$ is not differentiable at $x = 0$.

$\therefore$ Rolle's theorem is not applicable.

$$(c) f(x) = x^2 - x \text{ in } [0, 1]$$

$$f(0) = f(1) = 0$$

$$f'(x) = 2x - 1$$

$$\text{Put } f'(x) = 0 \Rightarrow x = \frac{1}{2} \text{ or } c = \frac{1}{2} \in [0, 1]$$

$\therefore$ Rolle's theorem is applicable.

$$(d) f(x) = [x] \text{ in } [2.5, 2.7]$$

Also $f(2.5) = f(2.7) = 2$ is continuous and

differentiable in $[2.5, 2.7]$

$\therefore$ Rolle's theorem is applicable.

$$48. (c): \text{ Here } f(x) = x^3 - 6x^2 + 9x + 10$$

$$\Rightarrow f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$$

$f(x)$ will be increasing if $f'(x) \geq 0 \Rightarrow 3(x-3)(x-1) \geq 0$

$$\Rightarrow x \leq 1, x \geq 3$$

$\therefore$ Function $f(x)$ increases when $x \in (-\infty, 1] \cup [3, \infty)$

49. (None of the options is correct): Let x be the side of an equilateral triangle and A be its area.

$$\text{Now } \frac{dx}{dt} = 4 \text{ cm/sec} \quad [\text{Given}]$$

$$\text{Also, } A = \frac{\sqrt{3}}{4} x^2 \therefore \frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot 2x \cdot \frac{dx}{dt} = \frac{\sqrt{3}}{2} x \cdot \frac{dx}{dt}$$

$$\therefore (i) \text{ When } x = 14 \text{ cm, } \frac{dA}{dt} = \frac{\sqrt{3}}{2} \times 14 \times 4 = 28\sqrt{3} \text{ cm}^2/\text{sec}$$

$$50. (a): \text{ Let } f(x) = \sqrt{x}$$

$$\therefore f'(x) = \frac{1}{2\sqrt{x}}$$

$$\text{Here, } x = 25 \text{ and } \delta x = -0.01$$

$$\text{Now, } f(x + \delta x) - f(x) = f'(x) \delta x$$

$$\Rightarrow f(24.99) = \frac{1}{2\sqrt{25}} (-0.01) + \sqrt{25} = 5 - 0.001 = 4.999$$

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51. (a): Let $I = \int_{-3}^3 \cot^{-1} x \, dx$

$$= \cot^{-1} x \int_{-3}^3 dx - \int_{-3}^3 \left[\frac{d}{dx} (\cot^{-1} x) \right] dx$$

$$= \left[\cot^{-1} x \cdot x \right]_{-3}^3 - \int_{-3}^3 \left(\frac{-1}{1+x^2} \right) \cdot x \, dx$$

$$= [3 \cot^{-1} 3 - (-3) \cot^{-1}(-3)] + \int_{-3}^3 \frac{x}{1+x^2} \, dx$$

$$= 3 \cot^{-1} 3 + 3 \cot^{-1}(-3) + \frac{1}{2} [\log(1+x^2)]_{-3}^3$$

$$= 3 \cot^{-1} 3 + 3(\pi - \cot^{-1} 3) + \frac{1}{2} (\log 10 - \log 10) = 3\pi$$

52. (d): Let $I = \int \frac{1}{\sqrt{x} + x\sqrt{x}} \, dx = \int \frac{1}{\sqrt{x}(1+(\sqrt{x})^2)} \, dx$

Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} \, dx = dt$

$$\therefore I = 2 \int \frac{dt}{1+t^2} = 2 \tan^{-1} t + C = 2 \tan^{-1} \sqrt{x} + C$$

53. (d): Let $I = \int \frac{2x-1}{(x-1)(x+2)(x-3)} \, dx$

Let $\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{P}{x-1} + \frac{Q}{x+2} + \frac{R}{x-3}$

$$\Rightarrow 2x-1 = P(x+2)(x-3) + Q(x-1)(x-3) + R(x-1)(x+2)$$

Putting $x = 1, 1 = P(3)(-2) \Rightarrow P = -\frac{1}{6}$

Putting $x = -2, -5 = Q(-3)(-5) \Rightarrow Q = -\frac{1}{3}$

Putting $x = 3, 5 = R(2)(5) \Rightarrow R = \frac{1}{2}$

$$\therefore I = \frac{-1}{6} \int \frac{dx}{x-1} - \frac{1}{3} \int \frac{dx}{x+2} + \frac{1}{2} \int \frac{dx}{x-3}$$

$$= -\frac{1}{6} \log |x-1| - \frac{1}{3} \log |x+2| + \frac{1}{2} \log |x-3| + K$$

$$\therefore A = -\frac{1}{6}, B = -\frac{1}{3}, C = \frac{1}{2}$$

54. (a): Let $I = \int_0^2 [x^2] \, dx$

$$= \int_0^1 [x^2] \, dx + \int_1^{\sqrt{2}} [x^2] \, dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2] \, dx + \int_{\sqrt{3}}^2 [x^2] \, dx$$

$$= 0 + \int_1^{\sqrt{2}} 1 \, dx + \int_{\sqrt{2}}^{\sqrt{3}} 2 \, dx + \int_{\sqrt{3}}^2 3 \, dx = [x]_1^{\sqrt{2}} + 2[x]_{\sqrt{2}}^{\sqrt{3}} + 3[x]_{\sqrt{3}}^2$$

$$= (\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3}) = -\sqrt{2} - \sqrt{3} + 5$$

55. (c): Let $I = \int_0^1 \frac{1+x}{\sqrt{1-x}} \, dx$

$$= \int_0^1 \frac{(1+x)^2}{\sqrt{1-x^2}} \, dx = \int_0^1 \frac{1+x}{\sqrt{1-x^2}} \, dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} \, dx + \int_0^1 \frac{x}{\sqrt{1-x^2}} \, dx$$

Put $1-x^2 = t \Rightarrow -2x \, dx = dt$

Now when $x = 0, t = 1$ and when $x = 1, t = 0$

$$\therefore I = [\sin^{-1} x]_0^1 - \frac{1}{2} \int_1^0 \frac{dt}{\sqrt{t}} = (\sin^{-1} 1 - \sin^{-1} 0) + \frac{1}{2} \int_0^1 \frac{dt}{\sqrt{t}}$$

$$= \frac{\pi}{2} + [\sqrt{t}]_0^1 = \frac{\pi}{2} + 1$$

56. (d): Given $n(U) = 100, n(A) = 50, n(B) = 60,$

$n(A \cap B) = 20$

Since $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore n(A \cup B) = 50 + 60 - 20 = 90$

Now, $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

$$= 100 - 90 = 10$$

57. (a): Given $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{x^2 - 7x + 12}$

Now, $f(x)$ will be defined if $x^2 - 7x + 12 \geq 0$

$\Rightarrow (x-4)(x-3) \geq 0 \Rightarrow x \in (-\infty, 3] \cup [4, \infty)$

$\therefore$ Domain of $f(x) = (-\infty, 3] \cup [4, \infty)$

58. (a): Here, $\cos x = |\sin x|$

$\Rightarrow \sin x = \pm \cos x \Rightarrow \tan x = \pm 1 = \pm \tan \frac{\pi}{4}$

$\Rightarrow x = n\pi \pm \left(\frac{\pi}{4}\right), n \in \mathbb{Z}$

59. (b): Consider $\sqrt{3} \operatorname{cosec} 20^\circ - \sec 20^\circ$

$$= \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3} \cos 20^\circ - \sin 20^\circ}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2 \left(\frac{\sqrt{3}}{2} \cos 20^\circ - \frac{1}{2} \sin 20^\circ \right)}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2(\sin 60^\circ \cos 20^\circ - \cos 60^\circ \sin 20^\circ)}{\sin 20^\circ \cos 20^\circ}$$

$$= \frac{2 \times 2 \sin(60^\circ - 20^\circ)}{2 \sin 20^\circ \cos 20^\circ} = \frac{4 \sin 40^\circ}{\sin 40^\circ} = 4$$

60. (b): Here $P(n) : 2^n < n!$

If $n = 2$, then $2^2 = 4$ and $2! = 2$, so $4 \not< 2$.

If $n = 3$, then $2^3 = 8$ and $3! = 6$, so $8 \not< 6$

If $n = 4$, then $2^4 = 16$ and $4! = 24$, so $16 < 24$

$\therefore$ The smallest positive integer for which $P(n)$ is true is 4.



OLYMPIAD PROBLEMS



ATHLON M. MAR

1. Find all triplets (x, y, z) of natural numbers such that y is a prime number, y and 3 do not divide z , and $x^3 - y^3 = z^2$.

2. Let B_1 and C_1 be points on the sides AC and AB of triangle ABC . Lines BB_1 and CC_1 intersect at point D . Prove that a circle can be inscribed inside quadrilateral AB_1DC_1 if and only if the incircles of the triangles ABD and ACD are tangent to each other.

3. Let a be a real number. Let $\{f_n(x)\}$ be a sequence of polynomials such that $f_0(x) = 1$ and $f_{n+1}(x) = xf_n(x) + f_n(ax)$ for $n = 0, 1, 2, \dots$

(a) Prove that $f_n(x) = x^n f_n\left(\frac{1}{x}\right)$ for $n = 0, 1, 2, \dots$

(b) Find an explicit expression for $f_n(x)$.

4. Given a 10×10 board, we want to remove n of the 100 squares so that no 4 of the remaining squares form the corners of a rectangle with sides parallel to the sides of the board. Determine the minimum value of n .

5. Let x, y, z be non-negative real numbers such that $x + y + z = 1$. Prove that $x^2y + y^2z + z^2x \leq \frac{4}{27}$, and determine when equality occurs.

6. A convex quadrilateral $ABCD$ is inscribed in a circle whose centre O is inside the quadrilateral. Let $MNPQ$ be the quadrilateral whose vertices are the projections of the intersection point of the diagonals AC and BD onto the sides of $ABCD$. Prove that $2[MNPQ] \leq [ABCD]$.

7. Find the number of all natural numbers n , $4 \leq n \leq 1023$, whose binary representations do not contain three consecutive equal digits.

8. Show that there exists a triangle ABC for which, with the usual labelling of sides and medians, it is true that $a \neq b$ and $a + m_a = b + m_b$. Show further that

there exists a number k such that for each such triangle $a + m_a = b + m_b = k(a + b)$. Finally, find all possible ratios $a : b$ of the sides of these triangles.

9. Find all pairs of real numbers a and b such that the system of equations

$$\frac{x+y}{x^2+y^2} = a, \quad \frac{x^3+y^3}{x^2+y^2} = b$$

has a solution in real numbers (x, y) .

10. In a tetrahedron $ABCD$ we denote by E and F the midpoints of the medians from the vertices A and D , respectively. (The median from a vertex of a tetrahedron is the segment connecting the vertex and the centroid of the opposite face.) Determine the ratio of the volumes of tetrahedra $BCEF$ and $ABCD$.

11. I have four children. The age in years of each child is a positive integer between 2 and 16 inclusive and all four ages are distinct. A year ago the square of the age of the oldest child was equal to the sum of the squares of the ages of the other three. In one year's time the sum of the squares of the ages of the oldest and the youngest children will be equal to the sum of the squares of the other two children. Decide whether this information is sufficient to determine their ages uniquely, and find all possibilities for their ages.

12. Let ABC be an acute triangle with $\angle C > \angle B$. Let D be a point on side BC such that $\angle ADB$ is obtuse, and let H be the orthocentre of triangle ABD . Suppose that F is a point inside triangle ABC and is on the circumcircle of triangle ABD . Prove that F is the orthocentre of triangle ABC if and only if both of the following are true: $HD \parallel CF$, and H is on the circumcircle of triangle ABC .

13. Solve the system of equations

$$(1 + 4^{2x-y}) \cdot 5^{1-2x+y} = 1 + 2^{2x-y+1}$$

$$y^3 + 4x + 1 + \ln(y^2 + 2x) = 0.$$

*

1

4



14. Let $ABCD$ be a convex cyclic quadrilateral. Prove that $|AB - CD| + |AD - BC| \geq 2|AC - BD|$.

15. Let $f(x)$ be a continuous function defined on $[0, 1]$ such that

- (i) $f(0) = f(1) = 0$;
(ii) $2f(x) + f(y) = 3f\left(\frac{2x+y}{3}\right)$ for all $x, y \in [0, 1]$.

Prove that $f(x) = 0$ for all $x \in [0, 1]$.

SOLUTIONS

1. We rewrite the equation in the form

$$(x - y)(x^2 + xy + y^2) = z^2.$$

Any common divisor of $x - y$ and $x^2 + xy + y^2$ also divides both z^2 and $(x^2 + xy + y^2) - (x + 2y)(x - y) = 3y^2$. Because z^2 and $3y^2$ are relatively prime by assumption, $x - y$ and $x^2 + xy + y^2$ must be relatively prime as well. Therefore, both $x - y$ and $x^2 + xy + y^2$ are perfect squares.

Writing $a = \sqrt{x - y}$, we have

$$x^2 + xy + y^2 = (a^2 + y)^2 + (a^2 + y)y + y^2 = a^4 + 3a^2y + 3y^2$$

$$\text{and } 4(x^2 + xy + y^2) = (2a^2 + 3y)^2 + 3y^2.$$

Writing $m = 2\sqrt{x^2 + xy + y^2}$ and $n = 2a^2 + 3y$, we have

$$m^2 = n^2 + 3y^2$$

or $(m - n)(m + n) = 3y^2$,

so $(m - n, m + n) = (1, 3y^2), (y, 3y)$ or $(3, y^2)$.

In the first case, $2n = 3y^2 - 1$ and $4a^2 = 2n - 6y = 3y^2 - 6y - 1$.

Hence, $a^2 \equiv 2 \pmod{3}$, which is impossible.

In the second case, $n = y < 2a^2 + 3y = n$, a contradiction.

In the third case, we have $4a^2 = 2n - 6y = y^2 - 6y - 3 < (y - 3)^2$.

When $y \geq 10$ we have $y^2 - 6y - 3 > (y - 4)^2$. Hence, we must actually have $y = 2, 3, 5$ or 7 . In this case

we have $a = \frac{\sqrt{y^2 - 6y - 3}}{2}$, which is real only when

$y = 7, a = 1, x = y + a^2 = 8$ and $z = 13$. This yields the unique solution $(x, y, z) = (8, 7, 13)$.

2. Let the incircle of triangle ABD be tangent to $\overline{AD}$ at T_1 and let the incircle of triangle ACD be tangent to $\overline{AD}$ at T_2 . Then

$$DT_1 = \frac{1}{2}(DA + DB - AB) \text{ and } DT_2 = \frac{1}{2}(DA + DC - AC)$$

First suppose a circle can be inscribed inside AB_1DC_1 .

Let it be tangent to sides AB_1, B_1D, DC_1, C_1A at points E, F, G, H , respectively. Using equal tangents, we have

$$AB - BD = (AH + HB) - (BF - DF)$$

$$= (AH + BF) - (BF - DF) = AH + DF$$

and similarly $AC - CD = AE + DG$. Also, $AH + DF = AE + DG$ by equal tangents, implying that $AB - BD = AC - CD$ and thus $DA + DB - AB = DA + DC - AC$. Therefore, $DT_1 = DT_2$, $T_1 = T_2$ and the two given incircles are tangent to each other.

Next suppose the two incircles are tangent to each other. Then $DA + DB - AB = DA + DC - AC$. Let ω be the incircle of ABB_1 and D' be the point on BB_1 (different from B_1) such that line CD' is tangent to ω . Suppose, by way of contradiction, that $D \neq D'$. From the result in the last paragraph, we know that the incircles of triangles ABD' and ACD' are tangent and hence $D'A + D'B - AB = D'A + D'C - AC$. Because $DB - AB = DC - AC$ and $D'B - AB = D'C - AC$, we must have $DB - D'B = DC - D'C$ by subtraction. Thus $DD' = |DB - D'B| = |DC - D'C|$. The triangle inequality then fails in triangle $DD'C$, a contradiction. This completes the proof.

3. When $a = 1$, we have $f_n(x) = (x + 1)^n$ for all n , and part (a) is easily checked. Now assume that $a \neq 1$.

Observe that f_n has degree n and always has constant term 1. Write $f_n(x) = c_0 + c_1x + \dots + c_nx^n$. We prove by induction on n that $(a^i - 1)c_i = (a^{n+1-i} - 1)c_{i-1}$ for $0 \leq i \leq n$ (where we let $c_{-1} = 0$).

The base case $n = 0$ is clear. Now suppose that $f_{n-1}(x) = b_0 + b_1x + \dots + b_{n-1}x^{n-1}$ satisfies the claim: specifically, we know $(a^i - 1)b_i = (a^{n-i} - 1)b_{i-1}$ and $(a^{n+1-i} - 1)b_{i-2} = (a^{i-1} - 1)b_{i-1}$ for $i \geq 1$.

For $i = 0$, the claim states $0 = 0$. For $i \geq 1$, the given recursion gives $c_i = b_{i-1}a^i b_i$ and $c_{i-1} = b_{i-2} + a^{i-1}b_{i-1}$. Then the claim is equivalent to

$$(a^i - 1)c_i = (a^{n+1-i} - 1)c_{i-1}$$

$$\Leftrightarrow (a^i - 1)(b_{i-1} + a^i b_i) = (a^{n+1-i} - 1)(b_{i-2} + a^{i-1}b_{i-1})$$

$$\Leftrightarrow (a^i - 1)b_{i-1} + a^i(a^i - 1)b_i = (a^{n+1-i} - 1)b_{i-2} + (a^n - a^{i-1})b_{i-1}$$

$$\Leftrightarrow (a^i - 1)b_{i-1} + a^i(a^{n-i} - 1)b_{i-1} = (a^{i-1} - 1)b_{i-1} + (a^n - a^{i-1})b_{i-1}$$

$$\Leftrightarrow (a^n - 1)b_{i-1} = (a^n - 1)b_{i-1}$$

So it is true.

Now by telescoping products, we have

$$c_i = \frac{c_i}{c_0} = \prod_{k=1}^i \frac{c_k}{c_{k-1}} = \prod_{k=1}^i \frac{a^{n+1-k} - 1}{a^k - 1} = \prod_{k=1}^i \frac{(a^{n+1-i} - 1)}{(a^k - 1)}$$

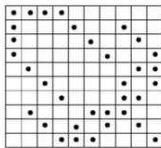
$$= \prod_{k=1}^i \frac{(a^k - 1)}{(a^k - 1)} = \prod_{k=1}^{n-i} \frac{a^{n+1-k} - 1}{a^k - 1}$$



$$= \prod_{k=1}^{n-i} \frac{c_k}{c_{k-1}} = \frac{c_{n-i}}{c_0} = c_{n-i}, \text{ giving our explicit form.}$$

Also, $f_n(x) = x^n f_n\left(\frac{1}{x}\right)$ if and only if $c_i = c_{n-i}$ for $i = 0, 1, \dots, n$, and from above this is indeed the case. This completes the proof.

4. The answer is 66. Consider the diagram, in which a dot represents a square that has not been removed. The diagram demonstrates that n can be 66:

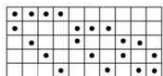


Now we proceed to show that n is at least 66.

Suppose, for sake of contradiction, that it were possible with $n = 65$. Denote by a_i the number of squares left in row i ($i = 1, 2, \dots, 10$). In row i , there are $\binom{a_i}{2}$ pairs of remaining squares. If no four remaining squares form the corners of a rectangle, then $N = \sum_{i=1}^{10} \binom{a_i}{2}$ must not exceed $\binom{10}{2} = 45$.

Note that, with a fixed $\sum_{i=1}^{10} a_i = 35$, the minimum of $\sum_{i=1}^{10} \binom{a_i}{2}$ is attained when and only when no two a_i 's differ by more than 1. Thus, $45 = \sum_{i=1}^{10} \binom{a_i}{2} \geq 5 \cdot \binom{4}{2} + 5 \cdot \binom{3}{2} = 45$,

i.e., this minimum is attained here, implying that five of the a_i 's equal 4 and rest equal 3. Then it is easy to see that aside from permutations of the row and columns, the first five rows of the board must be as follows:



We inspect this figure and notice that it is now impossible for another row to contain at least 3 remaining squares without forming the vertices of a rectangle with sides parallel to the sides of the board. This is a contradiction, because each of the remaining 5 rows is supposed to have 3 remaining squares. Thus, it is impossible for n to be less than 66, and we are done.

5. Assume without loss of generality that $x = \max\{x, y, z\}$. If $x \geq y \geq z$, then

$$x^2y + y^2z + z^2x \leq x^2y + y^2z + z^2x + z(xy + (x-y)(y-z)) \\ = (x+z)^2y = 4 \left(\frac{1}{2} - \frac{1}{2}y\right) \left(\frac{1}{2} - \frac{1}{2}y\right) y \leq \frac{4}{27},$$

where the last inequality follows from the AM-GM inequality. Equality occurs if and only if $z = 0$

(from the first inequality) and $y = \frac{1}{3}$, in which case

$$(x, y, z) = \left(\frac{2}{3}, \frac{1}{3}, 0\right). \text{ If } x \geq z \geq y, \text{ then}$$

$$x^2y + y^2z + z^2x = x^2z + z^2y + y^2x - (x-z)(z-y)(x-y) \\ \leq x^2z + z^2y + y^2x \leq \frac{4}{27},$$

where the second inequality is true from the result we proved for $x \geq y \geq z$ (except with y and z reversed). Equality holds in the first inequality only when two of x, y, z are equal, and in the second inequality only when $(x, z, y) = \left(\frac{2}{3}, \frac{1}{3}, 0\right)$. Because these conditions can't both be true, the inequality is actually strict in this case.

Therefore the inequality is indeed true, and equality holds when (x, y, z) equals $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$, $\left(\frac{1}{3}, 0, \frac{2}{3}\right)$, or $\left(0, \frac{2}{3}, \frac{1}{3}\right)$.

6. The result actually holds even when $ABCD$ is not cyclic. We begin by proving the following result:

Lemma: If XW is an altitude of triangle XYZ , then $\frac{XW}{YZ} \leq \frac{1}{2} \tan\left(\frac{\angle Y + \angle Z}{2}\right)$.

Proof: X lies on an arc of a circle determined by YZ and $\angle YXZ = 180^\circ - \angle Y - \angle Z$. Its distance from YZ is maximized when it is at the centre of this arc, which occurs when $\angle Y = \angle Z$. At this point, $\frac{XW}{YZ} = \frac{1}{2} \tan\left(\frac{\angle Y + \angle Z}{2}\right)$.

Suppose M, N, P, Q are on sides AB, BC, CD, DA , respectively. Also let T be the intersection of AC and BD .

Let $\alpha = \angle ADB$, $\beta = \angle BAC$, $\gamma = \angle CAD$, $\delta = \angle DBA$. From the lemma,

$$MT \leq \frac{1}{2} AB \cdot \tan\left(\frac{\beta + \delta}{2}\right) \text{ and } QT \leq \frac{1}{2} AD \cdot \tan\left(\frac{\alpha + \gamma}{2}\right).$$

Also, $\angle MTQ = 180^\circ - \angle QAM = 180^\circ - \angle DAB$.

$$\text{Thus } 2[MTQ] = MT \cdot QT \sin \angle MTQ \leq \frac{1}{4} \tan\left(\frac{\alpha + \gamma}{2}\right)$$

$$\tan\left(\frac{\beta + \delta}{2}\right) AB \cdot AD \sin \angle DAB$$





Because $\frac{\alpha + \gamma}{2} + \frac{\beta + \delta}{2} = 90^\circ$, this last expression exactly equals $\frac{1}{4} AB \cdot AD \sin \angle DAB = \frac{1}{2} [ABD]$.

Thus, $2[MTQ] \leq \frac{1}{2} [ABD]$.

Likewise, $2[NTM] \leq \frac{1}{2} [BCA]$, $[PTN] \leq \frac{1}{2} [CDB]$ and $[QTP] \leq \frac{1}{2} [DAC]$. Adding these four inequalities shows that $2[MNPQ]$ is at most $\frac{1}{2} ([ABD] + [CDB]) + \frac{1}{2} ([BCA] + [DAC]) = [ABCD]$, as desired.

7. A binary string is a finite string of digits, all either 0 or 1. Call such a string (perhaps starting with zeroes) valid if it does not contain three consecutive equal digits. Let s_n represent the number of valid n -digit strings; let s_n be the number of valid strings starting with two equal digits; and let d_n be the number of valid strings starting with two different digits. Observe that $a_n = s_n + d_n$ for all n .

An $(n+2)$ -digit string starting with 00 is valid if and only if its last n digits form a valid string starting with 1. Similarly, an $(n+2)$ -digit string starting with 11 is valid if and only if its last n digits form a valid string starting with 0. Thus, $s_{n+2} = a_n = s_n + d_n$.

An $(n+2)$ -digit string starting with 01 is valid if and only if its last n digits form a valid string starting with 01, 11, or 10. Thus, $d_{n+2} = s_n + 2d_n$. Solving these recursions gives $s_{n+4} = 3s_{n+2} - s_n$ and $d_{n+4} = 3d_{n+2} - d_n$, which when added together yield $a_{n+4} = 3a_{n+2} - a_n$.

Thus we can calculate initial values of a_n and then use the recursion to find other values.

n	1	2	3	4	5	6	7	8	9	10
a_n	2	4	6	10	16	26	42	68	110	178

Now of the a_n valid n -digit strings, only half start with 1. Thus only half are binary representations of positive numbers, and exactly

$$\frac{1}{2} (a_1 + a_2 + \dots + a_{10}) = 231$$

numbers between 1 and 1023 have the desired property. Ignoring 1, 2 and 3, we find that the answer is 228.

8. We know that

$$m_a^2 = \frac{1}{4} (2b^2 + 2c^2 - a^2), \quad m_b^2 = \frac{1}{4} (2a^2 + 2c^2 - b^2),$$

$$\text{so } m_a^2 - m_b^2 = \frac{3}{4} (b^2 - a^2)$$

If the desired condition $m_a - m_b = b - a \neq 0$ is to be satisfied, then necessarily $m_a + m_b = \frac{3}{4} (b + a)$. Find the system of equations

$$m_a - m_b = b - a$$

$$m_a + m_b = \frac{3}{4} (b + a)$$

we find that we would then have $m_a = \frac{1}{8} (7b - a)$,

$$m_b = \frac{1}{8} (7a - b), \text{ and } a + m_a = b + m_b = \frac{7}{8} (a + b).$$

$$\text{Thus } k = \frac{7}{8}.$$

Now we examine for what $a \neq b$ there exists a triangle ABC with sides a , b and medians

$m_a = \frac{1}{8} (7b - a)$, $m_b = \frac{1}{8} (7a - b)$. We can find all three side lengths of the triangle AB_1G , where G is the centroid of the triangle ABC and B_1 is the midpoint of the side AC :

$$AB_1 = \frac{b}{2}, \quad AG = \frac{2}{3} m_a = \frac{2}{3} \cdot \frac{1}{8} (7b - a) = \frac{1}{12} (7b - a),$$

$$B_1G = \frac{1}{3} m_b = \frac{1}{3} \cdot \frac{1}{8} (7a - b) = \frac{1}{24} (7a - b).$$

Examining the triangle inequalities for these three lengths, we get the condition $\frac{1}{3} < \frac{a}{b} < 3$, from which the value $\frac{a}{b} = 1$ has to be excluded by assumption.

This condition is also sufficient: once the triangle AB_1G has been constructed, it can always be completed to a triangle ABC with $b = AC$, $m_a = AA_1$, $m_b = BB_1$. Then from the equality $m_a^2 - m_b^2 = \frac{3}{4} (b^2 - a^2)$ we would also have $a = BC$.

9. If the given system has a solution (x, y) for $a = A$, $b = B$, then it clearly also has a solution (kx, ky) for $a = \frac{1}{k} A$, $b = kB$, for any $k \neq 0$. It follows that the existence of a solution of the given system depends only on the value of the product ab . We therefore begin by examining the values of the expression

$$P(u, v) = \frac{(u + v)(u^3 + v^3)}{(u^2 + v^2)^2}$$



where the numbers u and v are normalized by the condition $u^2 + v^2 = 1$.

This condition implies that

$$P(u, v) = (u + v)(u^3 + v^3) = (u + v)^2(u^2 - uv + v^2) \\ = (u^2 + 2uv + v^2)(1 - uv) = (1 + 2uv)(1 - uv).$$

Under the condition $u^2 + v^2 = 1$ the product uv can attain all values in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

(If $u = \cos \alpha$ and $v = \sin \alpha$, then $uv = \frac{1}{2} \sin 2\alpha$.)

Hence it suffices to find the range of values of the function $f(t) = (1 + 2t)(1 - t)$ on the interval $t \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.

From the formula $f(t) = -2t^2 + t + 1 = -2\left(t - \frac{1}{4}\right)^2 + \frac{9}{8}$

it follows that this range of values is the closed interval with endpoints $f\left(-\frac{1}{2}\right) = 0$ and $f\left(\frac{1}{4}\right) = \frac{9}{8}$.

This means that if the given system has a solution, its parameters a and b must satisfy $0 \leq ab \leq \frac{9}{8}$, where the equality $ab = 0$ is possible only if $x + y = 0$ (then, however, $a = b = 0$).

Conversely, if a and b satisfy $0 < ab \leq \frac{9}{8}$, by our proof there exist numbers u and v such that $u^2 + v^2 = 1$ and $(u + v)(u^3 + v^3) = ab$. Denoting $a' = u + v$ and $b' = u^3 + v^3$, the equality $a'b' = ab \neq 0$ implies that both ratios $\frac{a'}{a}$ and $\frac{b'}{b}$ have the same value $k \neq 0$. Then $(x, y) = (ku, kv)$ is clearly a solution of the given system for the parameter values a and b .

10. Let K and L be the midpoints of the edges BC and AD , and let A_0, D_0 be the centroids of triangles BCD and ABC , respectively. Both medians AA_0 and DD_0 lie in the plane AKD , and their intersection T (the centroid of the tetrahedron) divides them in ratio $3 : 1$. T is also the midpoint of $\overline{KL}$, because

$$\vec{T} = \frac{1}{4}(\vec{A} + \vec{B} + \vec{C} + \vec{D}) \\ = \frac{1}{2}\left(\frac{1}{2}(\vec{A} + \vec{D}) + \frac{1}{2}(\vec{B} + \vec{C})\right) = \frac{1}{2}(\vec{K} + \vec{L}).$$

It follows that $\frac{ET}{AT} = \frac{FT}{DT} = \frac{1}{3}$,

and hence $\triangle ATD \sim \triangle ETF$ and $EF = \frac{1}{3}AD$. Because

the plane BCL bisects both segments AD and EF into halves, it also divides both tetrahedra $ABCD$ and $BCEF$ into two parts of equal volume. Let G be the midpoint of EF . The corresponding volumes then satisfy

$$\frac{[BCEF]}{[ABCD]} = \frac{[BCGF]}{[BCLD]} = \frac{GF}{LD} \cdot \frac{[BCG]}{[BCL]} = \frac{1}{3} \cdot \frac{KG}{KL} = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}.$$

11. Let the children's present ages be $a + 1, b + 1, c + 1$ and $d + 1$.

We are given that $1 \leq a < b < c < d \leq 15$. Note that $b \leq 13$ so that $b - a \leq 12$. We are also given

$$d^2 = a^2 + b^2 + c^2 \quad \dots(i)$$

$$\text{and } (d + 2)^2 + (a + 2)^2 = (b + 2)^2 + (c + 2)^2 \quad \dots(ii)$$

$$\text{Subtracting (i) from (ii) gives } 4(a + d) + a^2 = 4(b + c) - a^2$$

$$\text{or } a^2 = 2(b + c - a - d) \quad \dots(iii)$$

Thus a must be even because its square is even.

Furthermore, because $d > c$,

$$a^2 = 2(b - a + (c - d)) < 2(b - a) < 24, \text{ and hence either}$$

$$a = 2 \text{ and } a = 4.$$

If $a = 4$ then, because $a^2 < 2(b - a)$, we have $2b > a^2 + 2a = 24$ so that $b > 12$. This forces $b = 13$, $c = 14$ and $d = 15$, which contradicts the given conditions.

Thus $a = 2$. Equation (iii) gives $b + c - d = 4$. Substituting $a = 2$ and $d = b + c - 4$ into (i) and simplifying yields $(b - 4)(c - 4) = 10 = 1 \cdot 10 = 2 \cdot 5$.

Therefore we have $(b, c) = (5, 14)$ or $(6, 9)$, in which cases $d = 15$ and $d = 11$ respectively.

Hence the only possible solutions are $(a, b, c, d) = (2, 5, 14, 15)$ or $(2, 6, 9, 11)$, and these indeed satisfy (i) and (ii). It follows that there is no unique solution, and it is not possible to determine the children's ages.

12. All angles are directed modulo 180° . First observe that if P is the orthocentre of triangle UVW , then $\angle VPW = (90^\circ - \angle WPV) + (90^\circ - \angle WVP) = \angle WV + \angle UV = 180^\circ - \angle VOW$. Suppose that F is the orthocentre of triangle ABC .

Then $\angle ACB = 180^\circ - \angle AFB = 180^\circ - \angle ADB = \angle AHB$, so $ACHB$ is cyclic. As for the other condition, lines CF and HD are both perpendicular to side AB , so they are parallel.

Conversely, suppose that $HD \parallel CF$ and that H is on the circumcircle of triangle ABC . Because $AFDB$ and $AHCB$ are cyclic,

$$\angle AFB = \angle ADB = 180^\circ - \angle AHB = 180^\circ - \angle ACB.$$

Thus, F is an intersection point of the circle defined by $\angle AFB = 180^\circ - \angle ACB$ and the line defined by $CF \perp AB$. There are only two such points: the orthocentre of triangle ABC and the reflection of C across line AB . The latter point lies outside of triangle





ABC , and hence F must indeed be the orthocentre of triangle ABC and the reflection of C across line AB . The latter point lies outside of triangle ABC , and hence F must indeed be the orthocentre of triangle ABC .

13. The only solution is $(x, y) = (0, -1)$, which yields $5 = 5$ and $0 = 0$ in the above equations.

We first solve the first equation for $t = 2x - y$. Multiplying the equation by 5^{t-1} yields

$$(1 - 5^{t-1}) + 4(4^{t-1} - 10^{t-1}) = 0.$$

If $t > 1$, then both $1 - 5^{t-1}$ and $4^{t-1} - 10^{t-1}$ are negative; if $t < 1$, then both of these terms are positive. Therefore, $2x - y = 1$.

Substituting $2x = y + 1$ into the second equation, we find that $y^2 + 2y + 3 + \ln(y^2 + y + 1) = 0$.

This has solution $y = -1$. To prove that this is the only solution, it suffices to show that $f(y) = y^2 + 2y + 3 + \ln(y^2 + y + 1)$ is always increasing.

The derivative of f is

$$f'(y) = 3y^2 + 2 + \frac{2y + 1}{y^2 + y + 1}.$$

Observe that $2(y^2 + y + 1) + (2y + 1) = 2(y + 1)^2 + 1 > 0$ and that

$$y^2 + y + 1 > \left(y + \frac{1}{2}\right)^2 \geq 0. \text{ Thus}$$

$$f'(y) = 3y^2 + \frac{2(y^2 + y + 1) + (2y + 1)}{y^2 + y + 1} > 3y^2 \geq 0$$

for all y , as desired.

14. Let E be the intersection of $\overline{AC}$ and $\overline{BD}$. Then the triangles ABE and DCE are similar, so if we let $x = AE$, $y = BE$, $z = AB$, then there exists k such that

$$kx = DE, ky = CE, kz = CB.$$

$$\text{Now } |AB - CD| = |(kx - 1)z$$

and $|AC - BD| = |(kx + y) - (ky + x)| = |k - 1| \cdot |x - y|$. Because $|x - y| \leq z$ by the triangle inequality, we conclude $|AB - CD| \geq |AC - BD|$, and similarly $|AD - BC| \geq |AB - BD|$. These two inequalities imply the desired result.

Alternative Solution: Let $2\alpha, 2\beta, 2\gamma, 2\delta$ be the measures of the arcs subtended by AB, BC, CD, DA , respectively, and take the radius of the circumcircle of $ABCD$ to be 1. Assume without loss of generality that $\beta \leq \delta$. Then $\alpha + \beta + \gamma + \delta = 180^\circ$, and (by the Extended Law of Sines)

$$|AB - CD| = 2 \left| \sin \alpha - \sin \gamma \right| = 4 \left| \sin \frac{\alpha - \gamma}{2} \right| \left| \cos \frac{\alpha + \gamma}{2} \right|$$

$$\text{and } |AC - BD| = 2 \left| \sin(\alpha + \beta) - \sin(\beta + \gamma) \right|$$

$$= 4 \left| \sin \frac{\alpha - \gamma}{2} \right| \left| \cos \left(\frac{\alpha + \gamma}{2} + \beta \right) \right|.$$

Because $0 \leq \frac{1}{2}(\alpha + \gamma) \leq \frac{1}{2}(\alpha + \gamma) + \beta \leq 90^\circ$ (by the assumption $\beta \leq \delta$) and the cosine function is nonnegative and decreasing on $[0, 90^\circ]$, we conclude that $|AB - CD| \geq |AC - BD|$ and similarly $|AD - BC| \geq |AC - BD|$.

15. We prove by induction on n that $f\left(\frac{m}{3^n}\right) = 0$ for all integers $n \geq 0$ and all integers $0 \leq m \leq 3^n$. The given conditions show that this claim is true for $n = 0$. Now assuming it is true for $n = k - 1 \geq 0$, we prove it is true for $n = k$.

$$\text{If } m \equiv 0 \pmod{3} \text{ then } f\left(\frac{m}{3^k}\right) = f\left(\frac{\frac{m}{3}}{3^{k-1}}\right) = 0$$

by the induction hypothesis.

If $m \equiv 1 \pmod{3}$, then $1 \leq m \leq 3^k - 2$ and

$$3f\left(\frac{m}{3^k}\right) = 2f\left(\frac{\frac{m-1}{3}}{3^{k-1}}\right) + f\left(\frac{\frac{m+2}{3}}{3^{k-1}}\right) = 0 + 0 = 0.$$

$$\text{Thus } f\left(\frac{m}{3^k}\right) = 0.$$

Finally, if $m \equiv 2 \pmod{3}$, then $2 \leq m \leq 3^k - 1$ and

$$3f\left(\frac{m}{3^k}\right) = 2f\left(\frac{\frac{m-1}{3}}{3^{k-1}}\right) + f\left(\frac{\frac{m-2}{3}}{3^{k-1}}\right) = 0 + 0 = 0.$$

$$\text{Hence } f\left(\frac{m}{3^k}\right) = 0, \text{ finishing our induction.}$$

Given any $x \in [0, 1]$, we can form a sequence of numbers of the form $\frac{m}{3^k}$ whose limit is x . Because $f(x)$ is continuous, it follows that $f(x) = 0$ for all $x \in [0, 1]$, as desired.

DO YOU KNOW ?

There are just four numbers (after 1) which are the sums of the cubes of their digits:

$$153 = 1^3 + 5^3 + 3^3$$

$$370 = 3^3 + 7^3 + 0^3$$

$$371 = 3^3 + 7^3 + 1^3$$

$$407 = 4^3 + 0^3 + 7^3$$



CBSE

warm-up!

CLASS-XII

Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 2

Inverse Trigonometric Functions

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

- All questions are compulsory.
- This question paper contains 36 questions.
- Question 1-20 in Section-A are very short-answer-objective type questions carrying 1 mark each.
- Question 21-26 in Section-B are short-answer type questions carrying 2 marks each.
- Question 27-32 in Section-C are long-answer-I type questions carrying 4 marks each.
- Question 33-36 in Section-D are long-answer-II type questions carrying 6 marks each.

SECTION - A

- Find the principal value of $\sec^{-1}(2)$.
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) 0
- Find the value of $\sin(\sin^{-1}x + \cos^{-1}x)$, $|x| \leq 1$.
(a) 1 (b) 0
(c) $\frac{1}{2}$ (d) none of these
- Evaluate : $\cos\left\{\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$
(a) 0 (b) -1 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$
- Simplify : $\tan^{-1}\left(\frac{a+bx}{b-ax}\right)$, $x < \frac{b}{a}$
(a) $\tan^{-1}\frac{a}{b} + \tan^{-1}x$ (b) $\tan^{-1}\frac{b}{a} + \tan^{-1}x$
(c) $\tan^{-1}\frac{a}{b}$ (d) none of these
- Domain of $\cos^{-1}[x]$ is
(a) $[-1, 2]$ (b) $[-1, 2)$
(c) $(-1, 2]$ (d) none of these
- If $\theta = \tan^{-1}a$, $\phi = \tan^{-1}b$ and $ab = -1$, then $(\theta - \phi)$ is equal to
(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{2}$ (d) none of these
- If $\tan^{-1}3 + \tan^{-1}x = \tan^{-1}8$, then $x =$
(a) 5 (b) $\frac{1}{5}$
(c) $\frac{5}{14}$ (d) $\frac{14}{5}$
- $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c-a}\right)$ is equal to
(a) 0 (b) $\frac{\pi}{4}$ (c) 1 (d) 5



9. The principal value of $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ is
 (a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

10. $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{8}{17} =$

- (a) $\sin^{-1}\left(\frac{85}{77}\right)$ (b) $\sin^{-1}\left(\frac{77}{85}\right)$
 (c) $\sin^{-1}\left(\frac{88}{75}\right)$ (d) none of these

11. Evaluate: $\operatorname{cosec}\left\{\cot^{-1}\left(-\frac{4}{3}\right)\right\}$

12. Evaluate: $\sin(2 \sin^{-1} 0.6)$

13. For the principal values, evaluate $\cot^{-1}(-1) + \operatorname{cosec}^{-1}(-\sqrt{2}) + \sec^{-1}(2)$.

14. Find the value of $\cot(\tan^{-1} a + \cot^{-1} a)$.

15. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then find the value of $(\cos^{-1} x + \cos^{-1} y)$.

16. Evaluate: $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$

17. Find the value of $\cot\left[\sin^{-1}\left\{\cos(\tan^{-1} 1)\right\}\right]$.

18. Prove that: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

19. Find the domain of $\sec^{-1}(2x + 1)$.

20. Find the value of $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$, $|x| \geq 1$.

SECTION-B

21. If $\cot^{-1}\left(-\frac{1}{5}\right) = x$, then find the values of $\sin x$ and $\cos x$.

22. Write the function $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$, $x > 1$ in the simplest form.

23. Find the minimum value of n for which $\tan^{-1} \frac{n}{\pi} > \frac{\pi}{4}$, $n \in \mathbb{N}$.

OR

Prove that: $\frac{1}{2} \tan^{-1} x = \cos^{-1}\left(\frac{1 + \sqrt{1+x^2}}{2\sqrt{1+x^2}}\right)$

24. Prove that: $\cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right) = \frac{\pi}{3}$

25. If $x, y, z \in [-1, 1]$ such that $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 0$, then find the value of $x + y + z$.

26. Find the value of $\tan^{-1}\left(\cos^{-1}\frac{\sqrt{5}}{3}\right)$.

OR

Simplify: $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$, $\frac{\pi}{4} < x < \frac{5\pi}{4}$

SECTION-C

27. Solve the following equation for x :

$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$

28. Prove that: $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$

$$+ \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right] = \frac{2b}{a}$$

OR

Prove that: $2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

29. Using principal values, find the value of

$$\cot^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)\right\}$$

30. If $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$, then prove that $\sin y = \tan^2 \frac{x}{2}$.

31. If $c_i > 0$ for $i = 1, 2, \dots, n$, prove that

$$\tan^{-1}\left(\frac{c_1 x - y}{c_1 y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2 c_1}\right) +$$

$$\tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3 c_2}\right) + \dots + \tan^{-1}\frac{1}{c_n} = \tan^{-1} \frac{x}{y}$$

OR

$$\text{If } \tan^{-1}\left(\frac{1}{1+1 \cdot 2}\right) + \tan^{-1}\left(\frac{1}{1+2 \cdot 3}\right) +$$

$$\dots + \tan^{-1}\left(\frac{1}{1+n \cdot (n+1)}\right) = \tan^{-1} \theta, \text{ then find the value of } \theta.$$

32. Solve for x : $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$

SECTION-D

33. If $a, b, c > 0$ and $s = \frac{a+b+c}{2}$, then prove that

$$\tan^{-1} \sqrt{\frac{2as}{bc}} + \tan^{-1} \sqrt{\frac{2bs}{ca}} + \tan^{-1} \sqrt{\frac{2cs}{ab}} = \pi.$$



OR

Solve for x : $\cos^{-1} x + \sin^{-1} \left(\frac{x}{2} \right) = \frac{\pi}{6}$

34. Prove that:

$$\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2}{3} \right)$$

35. Prove that:

$$\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \frac{1+x^2}{2+x^2}$$

OR

Prove that: $\tan^{-1} \left\{ \frac{\cos 2\alpha \sec 2\beta + \cos 2\beta \sec 2\alpha}{2} \right\}$
 $= \tan^{-1} \left\{ \tan^2(\alpha + \beta) \tan^2(\alpha - \beta) \right\} + \tan^{-1} 1$

36. Find the maximum and minimum values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$, where $-1 \leq x \leq 1$.

1. (c) : In $[0, \pi]$, $\sec^{-1}(2) = \sec^{-1} \left(\sec \frac{\pi}{3} \right) = \frac{\pi}{3}$

2. (a) : We have, $\sin(\sin^{-1} x + \cos^{-1} x) = \sin \frac{\pi}{2} = 1$

3. (b) : $\cos \left[\cos^{-1} \left(-\frac{3}{2} \right) + \frac{\pi}{6} \right]$
 $= \cos \left[\pi - \cos^{-1} \left(\frac{3}{2} \right) + \frac{\pi}{6} \right] = \cos \left[\pi - \frac{\pi}{6} + \pi \right]$
 $= \cos \pi = -1$

4. (a) : We have, $\tan^{-1} \left(\frac{a+bx}{b-ax} \right)$
 $= \tan^{-1} \left(\frac{a+x}{1-\frac{a}{b}x} \right) = \tan^{-1} \frac{a}{b} + \tan^{-1} x$

5. (b) : Since $0 \leq \cos^{-1} x \leq \pi$
 $\Rightarrow -1 \leq x \leq 1 \Rightarrow -1 \leq x < 2 \Rightarrow x \in [-1, 2)$

6. (c) : Given, $\theta = \tan^{-1} a$ and $\phi = \tan^{-1} b$ and $ab = -1$
 $\therefore \tan \theta \tan \phi = ab = -1 \Rightarrow \tan \theta = -\cot \phi$
 $\Rightarrow \tan \theta = \tan \left(\frac{\pi}{2} + \phi \right) \Rightarrow \theta - \phi = \frac{\pi}{2}$

7. (b) : We have, $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$
 $\Rightarrow \tan^{-1} x = \tan^{-1} \left(\frac{8-3}{1+24} \right)$
 $\Rightarrow \tan^{-1} x = \tan^{-1} \left(\frac{5}{25} \right) \Rightarrow x = \frac{1}{5}$

8. (a) : $\cot^{-1} \left(\frac{ab+1}{a-b} \right) + \cot^{-1} \left(\frac{bc+1}{b-c} \right) + \cot^{-1} \left(\frac{ca+1}{c-a} \right)$
 $= \tan^{-1} \left(\frac{a-b}{1+ab} \right) + \tan^{-1} \left(\frac{b-c}{1+bc} \right) + \tan^{-1} \left(\frac{c-a}{1+ca} \right)$
 $= \tan^{-1} a - \tan^{-1} b + \tan^{-1} b - \tan^{-1} c + \tan^{-1} c - \tan^{-1} a = 0$

9. (a) : Let $\operatorname{cosec}^{-1} \left(-\frac{2}{3} \right) = \theta \Rightarrow \operatorname{cosec} \theta = -\frac{2}{3}$
 $= -\operatorname{cosec} \frac{\pi}{3} = \operatorname{cosec} \left(-\frac{\pi}{3} \right)$
 $\Rightarrow \theta = -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

$\therefore$ Principal value of $\operatorname{cosec}^{-1} \left(-\frac{2}{3} \right)$ is $\left(-\frac{\pi}{3} \right)$.

10. (b) : We have, $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{8}{17}$

$$= \sin^{-1} \left\{ \frac{3}{5} \times \frac{8}{17} + \frac{8}{17} \times \frac{3}{5} \right\}$$

$$= \sin^{-1} \left\{ \frac{3}{5} \times \frac{15}{17} + \frac{8}{17} \times \frac{4}{5} \right\} = \sin^{-1} \left(\frac{77}{85} \right)$$

11. $\operatorname{cosec} \left\{ \cot^{-1} \left(-\frac{4}{3} \right) \right\} = \operatorname{cosec} \left(\pi - \cot^{-1} \frac{4}{3} \right)$
 $= \operatorname{cosec} \left(\cot^{-1} \frac{4}{3} \right) = \operatorname{cosec} \left(\operatorname{cosec}^{-1} \frac{5}{3} \right) = \frac{5}{3}$

12. Consider $\sin(2 \sin^{-1} 0.6)$

$$= \sin \left[\sin^{-1} \left\{ 2 \times 0.6 \times \sqrt{1 - (0.6)^2} \right\} \right] \left[\because -\frac{1}{2} \leq 0.6 \leq \frac{1}{2} \right]$$

$$= \sin(\sin^{-1} 0.96) = 0.96$$

13. $\cot^{-1}(-1) + \operatorname{cosec}^{-1}(-2) + \sec^{-1}(2)$
 $= \frac{3\pi}{4} - \frac{\pi}{4} + \frac{\pi}{3} = \frac{5\pi}{6}$

14. Since we know that, $\tan^{-1} a + \cot^{-1} a = \frac{\pi}{2}$

$$\therefore \cot(\tan^{-1} a + \cot^{-1} a) = \cot \frac{\pi}{2} = 0$$

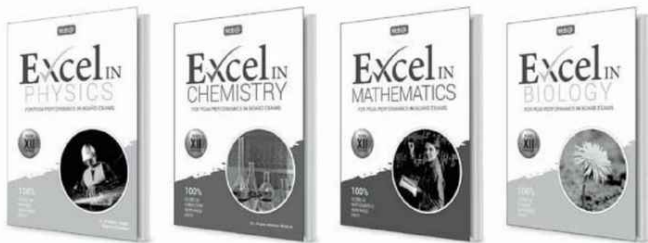
15. Given, $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x \right) + \left(\frac{\pi}{2} - \cos^{-1} y \right) = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1} x + \cos^{-1} y = \left(\pi - \frac{2\pi}{3} \right) = \frac{\pi}{3}$$



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- Comprehensive theory strictly based on NCERT, complemented with illustrations, activities and solutions of NCERT questions
- Practice questions & Model Test Papers for Board Exams
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16. $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) \neq \frac{3\pi}{4}$, because $\frac{3\pi}{4}$ does not lie

between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

$$\begin{aligned}\text{Now, } \tan^{-1}\left(\tan \frac{3\pi}{4}\right) &= \tan^{-1}\left\{\tan\left(\pi - \frac{\pi}{4}\right)\right\} \\ &= \tan^{-1}\left(-\tan \frac{\pi}{4}\right) = \tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\} = -\frac{\pi}{4}\end{aligned}$$

17. We have, $\cot[\sin^{-1}(\cos(\tan^{-1} 1))]$
 $= \cot\left\{\sin^{-1}\left(\cos \frac{\pi}{4}\right)\right\} = \cot\left\{\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\} = \cot \frac{\pi}{4} = 1$

18. Consider $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$
 $= \tan^{-1} 1 + (\tan^{-1} 2 + \tan^{-1} 3) = \tan^{-1} 1 + \tan^{-1}\left(\frac{2+3}{1-2 \cdot 3}\right)$
 $= \tan^{-1} 1 + \tan^{-1}(-1) = \frac{\pi}{4} + \frac{3\pi}{4} = \pi$

19. $\sec^{-1}(2x+1)$ is defined if $2x+1 \geq 1$ or $2x+1 \leq -1$
 $\Rightarrow 2x \geq 0$ or $2x \leq -2 \Rightarrow x \geq 0$ or $x \leq -1$
 $\Rightarrow x \in (-\infty, -1] \cup [0, \infty)$

Hence, the domain of $\sec^{-1}(2x+1)$ is $(-\infty, -1] \cup [0, \infty)$

20. Since we know that, $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$

$$\therefore \cos(\sec^{-1} x + \operatorname{cosec}^{-1} x) = \cos \frac{\pi}{2} = 0$$

21. Given, $\cot^{-1}\left(-\frac{1}{5}\right) = x$

$$\Rightarrow \cot x = -\frac{1}{5}, 0 < x < \pi$$

$$\therefore \operatorname{cosec} x = \sqrt{1 + \cot^2 x} = \sqrt{1 + \left(-\frac{1}{5}\right)^2} = \frac{\sqrt{26}}{5}$$

[$\because \operatorname{cosec} x$ is positive]

$$\Rightarrow \sin x = \frac{5}{\sqrt{26}}$$

$$\text{Also, } \cos x = \cot x \cdot \sin x = \left(-\frac{1}{5}\right) \frac{5}{\sqrt{26}} = -\frac{1}{\sqrt{26}}$$

22. Let $x = \sec y \Rightarrow y = \sec^{-1} x, x > 1, 0 < y < \frac{\pi}{2}$

$$\begin{aligned}\text{Now, } \sqrt{x^2 - 1} &= \sqrt{\sec^2 y - 1} = \sqrt{\tan^2 y} = |\tan y| = \tan y \\ \left(\because 0 < y < \frac{\pi}{2} \Rightarrow \tan y > 0 \Rightarrow |\tan y| = \tan y\right) \\ \therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right) &= \cot^{-1}\left(\frac{1}{\tan y}\right) = \cot^{-1}(\cot y) \\ &= y = \sec^{-1} x.\end{aligned}$$

23. We have, $\tan^{-1} \frac{n}{\pi} > \frac{\pi}{4} \Rightarrow \tan^{-1} \frac{n}{\pi} > \tan^{-1} 1$

$$\Rightarrow \tan\left(\tan^{-1} \frac{n}{\pi}\right) > \tan\left(\tan^{-1} 1\right)$$

[$\because \tan \theta$ is an increasing function]

$$\Rightarrow \frac{n}{\pi} > 1 \Rightarrow n > \pi \approx 3.14 \Rightarrow n = 4, 5, 6, \dots$$

Hence, the minimum value of n is 4.

OR

$$\text{Let } \tan^{-1} x = y \Rightarrow x = \tan y$$

$$\text{R. H. S.} = \cos^{-1}\left(\frac{1 + \sqrt{1 + \tan^2 y}}{\sqrt{2\sqrt{1 + \tan^2 y}}}\right) = \cos^{-1}\left(\sqrt{\frac{1 + \sec y}{2 \sec y}}\right)$$

$$\begin{aligned}&= \cos^{-1}\left(\sqrt{\frac{1 + \cos y}{2}}\right) = \cos^{-1}\left(\sqrt{\frac{2 \cos^2 \frac{y}{2}}{2}}\right) \\ &= \cos^{-1}\left(\cos \frac{y}{2}\right) = \frac{y}{2} = \frac{1}{2} \tan^{-1} x = \text{L. H. S.}\end{aligned}$$

24. Let $\cos^{-1} x = \theta$

$$\text{Then, } x = \cos \theta \text{ and } \sqrt{1 - x^2} = \sin \theta$$

$$\therefore \text{L. H. S.} = \cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3}}{2} \sqrt{1 - x^2}\right)$$

$$= \theta + \cos^{-1}\left(\cos \frac{\pi}{3} \cos \theta + \sin \frac{\pi}{3} \sin \theta\right)$$

$$= \theta + \cos^{-1}\left\{\cos\left(\frac{\pi}{3} - \theta\right)\right\} = \theta + \left(\frac{\pi}{3} - \theta\right) = \frac{\pi}{3} = \text{R. H. S.}$$

25. We have, $x, y, z \in [-1, 1]$

$$\Rightarrow -1 \leq x \leq 1, -1 \leq y \leq 1, -1 \leq z \leq 1$$

$$\Rightarrow 0 \leq \cos^{-1} x \leq \pi, 0 \leq \cos^{-1} y \leq \pi, 0 \leq \cos^{-1} z \leq \pi$$

$$\therefore \cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 0$$

[Given]

$$\Rightarrow \cos^{-1} x = 0, \cos^{-1} y = 0 \text{ and } \cos^{-1} z = 0$$

$$\Rightarrow x = y = z = 1. \text{ Hence, } x + y + z = 3.$$

26. Let $\cos^{-1} \frac{\sqrt{5}}{3} = \theta$. Then, $\cos \theta = \frac{\sqrt{5}}{3}$

$$\therefore \tan \frac{1}{2}\left(\cos^{-1} \frac{\sqrt{5}}{3}\right) = \tan \frac{\theta}{2} = \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}$$

$$= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \left[\frac{1 - \frac{\sqrt{5}}{3}}{1 + \frac{\sqrt{5}}{3}}\right]^{1/2} = \left[\frac{3 - \sqrt{5}}{3 + \sqrt{5}}\right]^{1/2}$$

$$= \left[\frac{3 - \sqrt{5}}{3 + \sqrt{5}} \times \frac{3 - \sqrt{5}}{3 - \sqrt{5}}\right]^{1/2} = \frac{(3 - \sqrt{5})}{2}.$$





OR

We have, $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right)$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x\right)$$

$$= \sin^{-1}\left\{\sin\left(x + \frac{\pi}{4}\right)\right\} = \pi - \left(x + \frac{\pi}{4}\right)$$

$$\left[\because \frac{\pi}{4} < x < \frac{5\pi}{4} \Rightarrow \frac{\pi}{2} < x + \frac{\pi}{4} < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} < \pi - \left(x + \frac{\pi}{4}\right) < \frac{\pi}{2}\right]$$

$$= \frac{3\pi}{4} - x$$

27. We have, $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{5}{x}\right)$$

$$\Rightarrow \frac{12}{x} = \sin\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{5}{x}\right)\right] = \cos\left[\sin^{-1}\left(\frac{5}{x}\right)\right]$$

$$\Rightarrow \frac{12}{x} = \sqrt{1 - \left(\frac{5}{x}\right)^2} \quad \left[\because \sin^{-1} x = \cos^{-1} \sqrt{1 - x^2}\right]$$

$$\Rightarrow \frac{144}{x^2} = 1 - \frac{25}{x^2} \Rightarrow \frac{144}{x^2} + \frac{25}{x^2} = 1$$

$$\Rightarrow x^2 = 169 \Rightarrow x = 13, -13.$$

But $x = -13$ does not satisfy the given equation.

Hence, the solution of the given equation is $x = 13$.

28. L.H.S. = $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\}$

$$\text{Let } \cos^{-1}\frac{a}{b} = \theta \Rightarrow \frac{a}{b} = \cos \theta$$

$$\therefore \text{L.H.S.} = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

$$= \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} + \frac{1 - \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2}} = \frac{1 + \tan^2 \frac{\theta}{2} + 1 + \tan^2 \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$= 2 \frac{\left(1 + \tan^2 \frac{\theta}{2}\right)}{\left(1 - \tan^2 \frac{\theta}{2}\right)} = \frac{2}{\cos\left(2 \cdot \frac{\theta}{2}\right)} = \frac{2}{\cos \theta} = \frac{2}{a/b} = \frac{2b}{a} = \text{R.H.S.}$$

OR

$$\text{L.H.S.} = 2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8}$$

$$= 2 \left\{ \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} \right\} + \sec^{-1} \frac{5\sqrt{2}}{7}$$

$$= 2 \tan^{-1} \left\{ \frac{\frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5} \times \frac{1}{8}} \right\} + \tan^{-1} \sqrt{\left(\frac{5\sqrt{2}}{7}\right)^2 - 1}$$

$$\left[\because \sec^{-1} x = \tan^{-1} \sqrt{x^2 - 1}\right]$$

$$= 2 \tan^{-1} \frac{13}{39} + \tan^{-1} \frac{1}{7} = 2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \left\{ \frac{2 \times \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2} \right\} + \tan^{-1} \frac{1}{7} \quad [\because |x| < 1]$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7} = \tan^{-1} \left\{ \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}} \right\} = \tan^{-1} 1 = \frac{\pi}{4} = \text{R.H.S.}$$

29. $\sqrt{\frac{2-\sqrt{3}}{4}} = \sqrt{\frac{4-2\sqrt{3}}{4 \times 2}} = \sqrt{\frac{(\sqrt{3}-1)^2}{2 \times 2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \sin \frac{\pi}{12}$

$$\therefore \sin^{-1} \sqrt{\frac{2-\sqrt{3}}{4}} = \sin^{-1} \left(\sin \frac{\pi}{12} \right) = \frac{\pi}{12}$$

$$\text{Also, } \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) = \cos^{-1} \left(\cos \frac{\pi}{6} \right) = \frac{\pi}{6}$$

$$\text{and } \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = \cos^{-1} \left(\cos \frac{\pi}{4} \right) = \frac{\pi}{4}$$

$$\therefore \cot^{-1} \left(\cot \left(\sin^{-1} \sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) + \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) \right) \right)$$

$$= \cot^{-1} \left(\cot \left(\frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4} \right) \right) = \cot^{-1} \left(\cot \left(\frac{\pi}{2} \right) \right) = \frac{\pi}{2}$$

30. Given, $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$$\Rightarrow y = \left(\frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) \right) - \tan^{-1}(\sqrt{\cos x})$$

$$\Rightarrow y = \frac{\pi}{2} - 2 \tan^{-1}(\sqrt{\cos x})$$

$$\Rightarrow y = \frac{\pi}{2} - \cos^{-1} \left(\frac{1 - \cos x}{1 + \cos x} \right) \Rightarrow y = \frac{\pi}{2} - \cos^{-1} \left(\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$

Monthly Test Drive-1 CLASS XI ANSWER KEY

1. (d) 2. (a) 3. (b) 4. (c) 5. (b)

6. (b) 7. (d) 8. (a,b) 9. (c,d) 10. (c,d)

11. (d) 12. (c) 13. (c,d) 14. (a) 15. (a)

16. (c) 17. (8001) 18. (10) 19. (219) 20. (168)



$$\Rightarrow y = \frac{\pi}{2} - \cos^{-1}\left(\tan^2 \frac{x}{2}\right) \Rightarrow \cos^{-1}\left(\tan^2 \frac{x}{2}\right) = \frac{\pi}{2} - y$$

$$\Rightarrow \tan^2 \frac{x}{2} = \cos\left(\frac{\pi}{2} - y\right) \Rightarrow \tan^2 \frac{x}{2} = \sin y.$$

$$\begin{aligned} 31. \text{ L.H.S.} &= \tan^{-1}\left(\frac{c_1 x - y}{c_1 y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_1 c_2}\right) + \dots + \tan^{-1}\frac{1}{c_n} \\ &= \tan^{-1}\left(\frac{\frac{x}{1} - \frac{1}{c_1}}{1 + \frac{x}{y} \cdot \frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}}\right) + \dots + \tan^{-1}\frac{1}{c_n} \\ &= \left(\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1}\right) + \left(\tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2}\right) \\ &\quad + \left(\tan^{-1}\frac{1}{c_2} - \tan^{-1}\frac{1}{c_3}\right) + \dots + \left(\tan^{-1}\frac{1}{c_{n-1}} - \tan^{-1}\frac{1}{c_n}\right) + \tan^{-1}\frac{1}{c_n} \\ &= \tan^{-1}\frac{x}{y} = \text{R.H.S.} \end{aligned}$$

OR

Refer to answer 47, Page no. 25 of MTG CBSE Champion Mathematics, Class-12

$$32. \text{ We have, } 2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$$

$$\Rightarrow 2 \tan^{-1}(\cos x) - \tan^{-1}(2 \operatorname{cosec} x) = 0$$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{1 - \cos^2 x}\right) - \tan^{-1}(2 \operatorname{cosec} x) = 0$$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x}{\sin^2 x}\right) - \tan^{-1}(2 \operatorname{cosec} x) = 0$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{2 \cos x}{\sin^2 x} - \frac{2}{\sin x}}{1 + \left(\frac{2 \cos x}{\sin^2 x}\right)\left(\frac{2}{\sin x}\right)}\right) = 0$$

$$\Rightarrow \tan^{-1}\left(\frac{2 \cos x \sin x - 2 \sin^2 x}{\sin^3 x + 4 \cos x}\right) = 0$$

$$\Rightarrow \frac{2 \cos x \sin x - 2 \sin^2 x}{\sin^3 x + 4 \cos x} = 0$$

$$\Rightarrow 2 \cos x \sin x = 2 \sin^2 x \Rightarrow \tan x = 1$$

$$\Rightarrow x = \tan^{-1}(1) = \tan^{-1}\left(\tan \frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$33. \text{ Consider } \sqrt{\frac{2as}{bc}} \times \sqrt{\frac{2bs}{ca}} = \sqrt{\frac{4abs^2}{abc^2}} = \frac{2s}{c} = \frac{a+b+c}{c}$$

$$\Rightarrow \sqrt{\frac{2as}{bc}} \times \sqrt{\frac{2bs}{ca}} = 1 + \frac{a+b}{c} > 1 \quad \left\{ \because \frac{a+b}{c} > 0 \right\}$$

$$\begin{aligned} \therefore \tan^{-1}\sqrt{\frac{2as}{bc}} + \tan^{-1}\sqrt{\frac{2bs}{ca}} + \tan^{-1}\sqrt{\frac{2cs}{ab}} \\ = \pi + \tan^{-1}\left\{\frac{\sqrt{\frac{2as}{bc}} + \sqrt{\frac{2bs}{ca}}}{1 - \sqrt{\frac{2as}{bc}} \times \sqrt{\frac{2bs}{ca}}}\right\} + \tan^{-1}\sqrt{\frac{2cs}{ab}} \\ = \pi + \tan^{-1}\left\{\frac{\sqrt{\frac{2s}{abc}}(a+b)}{1 - \frac{a+b+c}{c}}\right\} + \tan^{-1}\sqrt{\frac{2cs}{ab}} \\ = \pi + \tan^{-1}\left(\frac{\sqrt{\frac{2cs}{ab}}}{1}\right) + \tan^{-1}\sqrt{\frac{2cs}{ab}} \\ = \pi - \tan^{-1}\sqrt{\frac{2cs}{ab}} + \tan^{-1}\sqrt{\frac{2cs}{ab}} = \pi \end{aligned}$$

OR

Refer to answer 83, Page no. 30 of MTG CBSE Champion Mathematics, Class-12

34. We have to show that

$$\frac{9\pi}{8} - \frac{9}{4} \sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4} \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\text{or } \frac{\pi}{2} = \sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$\text{R.H.S.} = \sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

$$= \sin^{-1}\left(\frac{1}{3} \sqrt{1 - \left(\frac{2\sqrt{2}}{3}\right)^2} + \frac{2\sqrt{2}}{3} \sqrt{1 - \left(\frac{1}{3}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{1}{3} \cdot \frac{1}{3} + \frac{2\sqrt{2}}{3} \cdot \frac{2\sqrt{2}}{3}\right) = \sin^{-1}\left(\frac{1}{9} + \frac{8}{9}\right) = \sin^{-1}(1)$$

$$= \frac{\pi}{2} = \text{L.H.S.}$$

Hence, the given result is true.

35. Refer to answer 78, Page no. 29 of MTG CBSE Champion Mathematics, Class-12

Monthly Test Drive-1 CLASS XII ANSWER KEY

1. (c) 2. (b) 3. (a) 4. (b) 5. (b)
6. (b) 7. (a) 8. (a,c) 9. (a,b,c) 10. (a,b,c)
11. (a,b,c) 12. (b) 13. (a,d) 14. (c) 15. (d)
16. (a) 17. (360) 18. (2) 19. (3) 20. (6)



OR

$$\text{R.H.S.} = \tan^{-1} \{ \tan^2(\alpha + \beta) \tan^2(\alpha - \beta) \} + \tan^{-1} 1$$

$$= \tan^{-1} \left\{ \frac{\tan^2(\alpha + \beta) \tan^2(\alpha - \beta) + 1}{1 - \tan^2(\alpha + \beta) \tan^2(\alpha - \beta)} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sin^2(\alpha + \beta) \sin^2(\alpha - \beta) + \cos^2(\alpha + \beta) \cos^2(\alpha - \beta)}{\cos^2(\alpha + \beta) \cos^2(\alpha - \beta) - \sin^2(\alpha + \beta) \sin^2(\alpha - \beta)} \right\}$$

$$= \tan^{-1} \left\{ \frac{\{2 \sin(\alpha + \beta) \sin(\alpha - \beta)\}^2 + \{2 \cos(\alpha + \beta) \cos(\alpha - \beta)\}^2}{\{2 \cos(\alpha + \beta) \cos(\alpha - \beta)\}^2 - \{2 \sin(\alpha + \beta) \sin(\alpha - \beta)\}^2} \right\}$$

$$= \tan^{-1} \left\{ \frac{(\cos 2\beta - \cos 2\alpha)^2 + (\cos 2\alpha + \cos 2\beta)^2}{(\cos 2\alpha + \cos 2\beta)^2 - (\cos 2\beta - \cos 2\alpha)^2} \right\}$$

$$= \tan^{-1} \left\{ \frac{\cos^2 2\alpha + \cos^2 2\beta}{2 \cos 2\alpha \cos 2\beta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\cos 2\alpha \sec 2\beta + \cos 2\beta \sec 2\alpha}{2} \right\} = \text{L.H.S.}$$

36. Let $y = (\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$$\Rightarrow y = (\sin^{-1} x + \cos^{-1} x)^3 - 3 \sin^{-1} x \cos^{-1} x (\sin^{-1} x + \cos^{-1} x)$$

$$\Rightarrow y = \left(\frac{\pi}{2}\right)^3 - \frac{3\pi}{2} \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$\Rightarrow y = \frac{\pi^3}{8} - \frac{3\pi^2}{4} \sin^{-1} x + \frac{3\pi}{2} (\sin^{-1} x)^2$$

$$\Rightarrow \frac{3\pi}{2} (\sin^{-1} x)^2 - \frac{3\pi^2}{4} (\sin^{-1} x) + \left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow (\sin^{-1} x)^2 - \frac{\pi}{2} (\sin^{-1} x) + \frac{2}{3\pi} \left(\frac{\pi^3}{8} - y\right) = 0$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16} + \frac{\pi^2}{12} - \frac{2y}{3\pi} = 0$$

$$\Rightarrow \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 = \frac{2y}{3\pi} - \frac{\pi^2}{48} \quad \dots(i)$$

We know that, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ for all $x \in [-1, 1]$

$$\Rightarrow -\frac{3\pi}{4} \leq \sin^{-1} x - \frac{\pi}{4} \leq \frac{\pi}{4}$$

$$\Rightarrow 0 \leq \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \frac{9\pi^2}{16} \quad \dots(ii)$$

From (i) and (ii), we find that $0 \leq \frac{2y}{3\pi} - \frac{\pi^2}{48} \leq \frac{9\pi^2}{16}$

$$\Rightarrow \frac{\pi^2}{48} \leq \frac{2y}{3\pi} \leq \frac{9\pi^2}{16} + \frac{\pi^2}{48} \Rightarrow \frac{\pi^3}{32} \leq y \leq \frac{7\pi^3}{8}$$

Hence, the maximum and minimum values of

$$(\sin^{-1} x)^3 + (\cos^{-1} x)^3 \text{ are } \frac{7\pi^3}{8} \text{ and } \frac{\pi^3}{32} \text{ respectively.}$$

PUZZLE CORNER

MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. It's an overlapping, 2 in 1 mathdoku which is essentially in a 20 Doku layout.

In this puzzle 9 × 9 grid is given, your objective is to fill the digits 1-9 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

17+	4+		6+	17+	16+	1-		
		38+				3×		
			162×		5+		25+	
0-						3-	0-	29+
14×	0-		8+	1208×		7×		
	72×		1-			21+	1+	
		9+	27×	24×	0-		24+	9+
1-	13+	25+	6-	1-		2×		48×
					30×		16+	
					1-	4-	8×	2×
					45×		17+	4-
						10+		48×



Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1: Relations and Functions

Time Taken : 60 Min.

Only One Option Correct Type

- Let $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ be a relation on the set $A = \{1, 2, 3, 4\}$. The relation R is
(a) a function (b) transitive
(c) not symmetric (d) reflexive
- $f: R \rightarrow R$ given by $f(x) = 2x + |\cos x|$, is
(a) one-one and into (b) one-one and onto
(c) many-one and into (d) many-one and onto
- If $*$ is defined on the set R of all real numbers by $a * b = \sqrt{a^2 + b^2}$, then which of the following is true?
(a) 0 is the identity element in R w.r.t. $*$
(b) inverse of an element a in R w.r.t. $*$ is $1/b$, $b \neq 0$
(c) $*$ is not commutative
(d) none of these
- Let R be the real line. Consider the following subsets of the plane $R \times R$:
 $S = \{(x, y) : y = x + 1 \text{ and } 0 < x < 2\}$,
 $T = \{(x, y) : x - y \text{ is an integer}\}$.
Which one of the following is true?
(a) S is an equivalence relation on R but T is not.
(b) T is an equivalence relation on R but S is not.
(c) Neither S nor T is an equivalence relation on R .
(d) Both S and T are equivalence relations on R .
- Let $f(x) = \log_x 25$ and $g(x) = \log_x 5$. Then, $f(x) = g(x)$ holds for x belonging to
(a) R (b) $\{x : 0 < x < \infty, x \neq 1\}$
(c) ϕ (d) none of these
- If $f: [2, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x^2 - 4x^2}$, then $f^{-1}(x)$ is

- $\sqrt{2 + \sqrt{4 - \log_2 x}}$
- $\sqrt{2 + \sqrt{4 + \log_2 x}}$
- $\sqrt{2 - \sqrt{4 \log_2 x}}$
- $\sqrt{2 + \sqrt{4 \log_2 x}}$

One or More Than One Option(s) Correct Type

- Let $A = \{x \in R : x \geq 1/2\}$ and $B = \{x \in R : x \geq 3/4\}$. If $f: A \rightarrow B$ is defined as $f(x) = x^2 - x + 1$, then the solution set of the equation $f(x) = f^{-1}(x)$ is
(a) $\{1\}$ (b) $\{2\}$
(c) $\{1/2\}$ (d) none of these
- Domain of $f(x) = \sin^{-1}[2 - 4x^2]$ is $\{ \cdot \}$ denotes the greatest integer function)
(a) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right] - \{0\}$ (b) $\left[-\frac{\sqrt{3}}{2}, 0\right)$
(c) $\left[-\frac{\sqrt{3}}{2}, 0\right) \cup \left(0, \frac{\sqrt{3}}{2}\right]$ (d) $\left[-\frac{\sqrt{3}}{2}, 8\right]$
- Let f be the greatest integer function and g be the modulus function, then
(a) $(g \circ f - f \circ g)\left(-\frac{5}{3}\right) = 1$ (b) $(f + 2g)(-1) = 1$
(c) $(g \circ f - f \circ g)\left(\frac{5}{3}\right) = 0$ (d) $(f + 2g)(1) = 1$

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10. Let $f(x) = 2x - \sin x$ and $g(x) = \sqrt[3]{x}$, then
 (a) range of $g \circ f$ is R (b) $g \circ f$ is one-one
 (c) both f and g are one-one
 (d) both f and g are not one-one
11. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then, which of the following is true?
 (a) the identity element in A is $(1, 0)$
 (b) the inverse of (a, b) is $\left(\frac{1}{a}, -\frac{b}{a}\right)$ if $a \neq 0$
 (c) $*$ is not commutative
 (d) none of these
12. If $f(x) = \sin^2 x + \sin^2\left(x + \frac{\pi}{3}\right) + \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$ and $g(5/4) = 1$, then $(g \circ f)$ is
 (a) a polynomial of the first degree in $\sin x, \cos x$
 (b) a constant function
 (c) a polynomial of the second degree in $\sin x, \cos x$
 (d) none of these
13. Let $f(x) = \sec^{-1}[1 + \cos^2 x]$, where $[\cdot]$ denotes the greatest integer function, then
 (a) the domain of f is R
 (b) the domain of f is $[1, 2]$
 (c) the range of f is $[1, 2]$
 (d) the range of f is $[\sec^{-1} 1, \sec^{-1} 2]$

Comprehension Type

Here $a_0 = x$, $a_{n+1} = f(a_n)$, where $n = 0, 1, 2, \dots$

14. If $f: R \rightarrow R$ be given by $f(x) = 3 + 4x$ and $a_n = A + Bx$, then which of the following is not true?
 (a) $A + B + 1 = 2^{2n+1}$ (b) $|A - B| = 1$
 (c) $\lim_{n \rightarrow \infty} \frac{A}{B} = -1$ (d) none of these
15. If $f(x) = \frac{1}{1-x}$, then which of the following is not true?
 (a) $a_n = \frac{1}{1-x}$, if $n = 3k + 1$

- (b) $a_n = \frac{x-1}{x}$, if $n = 3k + 2$
 (c) $a_n = x$, if $n = 3k$ (d) none of these

Matrix Match Type

16. Match the following:

Column-I		Column-II	
P.	$f: R \rightarrow \left[\frac{3\pi}{4}, \pi\right)$ and $f(x) = \cot^{-1}(2x - x^2 - 2)$, then $f(x)$ is	1.	one-one
Q.	$f: R \rightarrow R$ and $f(x) = e^{px} \sin qx$, where $p, q \in R^+$, then $f(x)$ is	2.	into
R.	$f: R^+ \rightarrow [4, \infty)$ and $f(x) = 4 + 3x^2$, then $f(x)$ is	3.	many-one
S.	$f: X \rightarrow X$ and $f(f(x)) = x \forall x \in X$, then $f(x)$ is	4.	onto

P	Q	R	S
(a) 3, 4	3, 4	1, 2	1, 4
(b) 1, 4	1, 2	3, 4	2, 3
(c) 3, 4	1, 2	3, 4	1, 4
(d) 1, 4	3, 4	1, 2	3, 4

Numerical Answer Type

17. Let $n(A) = 4$ and $n(B) = 6$, then the number of one-one functions from A to B must be
18. Number of integers in the domain of function $f(x) + f(x^{-1}) = \frac{x^3 + 1}{x}$ is
19. If the number of different reflexive relations on a set A is equal to the number of different symmetric relations on set A , then the number of elements in set A is
20. Let $A = \{2, 3, 4, 5, \dots, 17, 18\}$. Let ' R ' be the equivalence relation on $A \times A$, cartesian product of A with itself, defined by $(a, b) R (c, d)$ iff $ad = bc$. Then, the number of ordered pairs of the equivalence class of $(3, 2)$ is

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SELF CHECK

No. of questions attempted
 No. of questions correct
 Marks scored in percentage

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.



CBSE

warm-up!

CLASS-XI

Synopsis and Chapterwise Practice questions for CBSE Exams as per the latest pattern and marking scheme issued by CBSE for the academic session 2019-20.

Series 2

Trigonometric Functions and Principle of Mathematical Induction

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

- All questions are compulsory.
- This question paper contains 36 questions.
- Question 1-20 in Section-A are very short-answer-objective type questions carrying 1 mark each.
- Question 21-26 in Section-B are short-answer type questions carrying 2 marks each.
- Question 27-32 in Section-C are long-answer-I type questions carrying 4 marks each.
- Question 33-36 in Section-D are long-answer-II type questions carrying 6 marks each.

SECTION - A

- Let $P(n) : "n^2 - n + 41 \text{ is a prime number}"$, then
 - $P(1)$ is not true
 - $P(3)$ is not true
 - $P(5)$ is not true
 - $P(41)$ is not true
- Find the value of $2\sin 15^\circ \cos 75^\circ$.
 - $\frac{(2+\sqrt{3})}{2}$
 - 1
 - $\frac{\sqrt{3}}{2}$
 - $\frac{2-\sqrt{3}}{2}$
- The principal solution of $\sin x = -\frac{1}{2}$ is
 - $\frac{7\pi}{6}$
 - $-\frac{7\pi}{6}$
 - $\frac{\pi}{6}$
 - $\frac{5\pi}{6}$
- The value of $\cos(35^\circ + A) \cos(35^\circ - B) + \sin(35^\circ + A) \sin(35^\circ - B)$ is equal to
 - $\sin(A+B)$
 - $\sin(A-B)$
 - $\cos(A+B)$
 - $\cos(A-B)$
- For all positive integral values of n , $3^{2n} - 2n + 1$ is divisible by
 - 2
 - 3
 - 5
 - none of these
- The value of $\tan 840^\circ$ is equal to
 - $\frac{1}{\sqrt{3}}$
 - $-\frac{1}{\sqrt{3}}$
 - $\sqrt{3}$
 - $-\sqrt{3}$
- $5 \sin \theta \cdot \sin 8\theta =$
 - $\frac{3}{2}(\cos 7\theta - \cos \theta)$
 - $\frac{5}{2}(\cos 9\theta - \cos 7\theta)$
 - $\frac{5}{2}(\cos 7\theta - \cos 9\theta)$
 - none of these
- In $\triangle ABC$, given $\angle A = 45^\circ$, $\angle B = 105^\circ$, $c = \sqrt{2}$, then
 - $b = \sqrt{3}$, $\angle C = 30^\circ$
 - $b = \sqrt{3} + 1$, $\angle C = 30^\circ$
 - $b = \sqrt{2}$, $\angle C = 30^\circ$
 - $b = \sqrt{3} - 1$, $\angle C = 30^\circ$
- For natural number n , $2^n (n-1)! < n^n$, if
 - $n < 2$
 - $n > 2$
 - $n \geq 2$
 - $n > 3$
- In a $\triangle ABC$, if $a = 2$, $b = 3$ and $\sin A = \frac{2}{3}$, then $\angle B$ is
 - 90°
 - 80°
 - 110°
 - 140°



11. Find the sign of $\sec 2000^\circ$.
 12. Find the value of $\cos (-1710^\circ)$.
 13. Find the range of $f(x) = 1 + 3 \cos 2x$.
 14. Solve $3 \cot^2 x - 1 = 0$.
 15. Find the degree measure corresponding to the radian measure $\left(\frac{11}{16}\right)^c$.

16. Find the value of $\cot \left(-\frac{15\pi}{4}\right)$.

17. Prove that $\cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$.

18. If three angles A , B and C are in A.P., then prove that $\cot B = \frac{\sin A - \sin C}{\cos C - \cos A}$.

19. Find the general solution of the equation $\tan^3 x - 3 \tan x = 0$.

20. Find the value of $\tan \frac{13\pi}{12}$.

SECTION - B

21. Use induction to prove the rule of exponents $(ab)^n = a^n b^n$ for all natural numbers n .
 22. Taking the moon's distance from the earth as 360000 km and the angle subtended by the moon at any point O on the earth as half a degree, estimate the diameter of the moon. (use $\pi = 3.1416$)

OR

If the angles of a triangle are in the ratio 3 : 4 : 5, find the smallest angle in degrees and the greatest angle in radians.

23. If $a \cos \theta - b \sin \theta = c$, then show that $a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$.
 24. If $\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$, then prove that $1 + \cot \alpha \tan \beta = 0$.

OR

Solve : $\sec x - 1 = (\sqrt{2} - 1) \tan x$

25. If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$, then prove that $\cos 2\alpha + \cos 2\beta + 2 \cos(\alpha + \beta) = 0$.
 26. If $\sin \theta = n \sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha) = \left(\frac{1+n}{1-n}\right) \tan \alpha$.

SECTION - C

27. If $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$, then prove that

$$\cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}.$$

28. Use mathematical induction to prove that $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! = (n+1)! - 1$ for all $n \in \mathbb{N}$.

OR

Prove by the principle of mathematical induction : $(n+3)^2 \leq 2^{n+3}$, $n \in \mathbb{N}$

29. If $\tan \theta = \frac{m}{n}$ and $\theta = 3\phi$ $\left(0 < \theta < \frac{\pi}{2}\right)$, then show that $\frac{m}{\sin \phi} - \frac{n}{\cos \phi} = 2\sqrt{m^2 + n^2}$.

30. If α and β satisfy the equation $a \tan \theta + b \sec \theta = c$, then show that $\tan(\alpha + \beta) = \frac{2ac}{a^2 - c^2}$.

OR

If $f(x) = \frac{\cot x}{1 + \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$, then find the value of $f(\alpha)f(\beta)$.

31. Prove that : $\cos 5A = 16 \cos^5 A - 20 \cos^3 A + 5 \cos A$
 32. If $0 < x < \frac{\pi}{2}$, then prove that $\cos(\sin x) > \sin(\cos x)$.

SECTION - D

33. Use induction to prove that

$$7 + 77 + 777 + \dots \text{ to } n \text{ terms} = \frac{7}{81}(10^{n+1} - 9n - 10),$$

for all $n \in \mathbb{N}$.

OR

Use induction to show that for all $n \in \mathbb{N}$

$$\sqrt{a} + \sqrt{a} + \sqrt{a} + \sqrt{a} + \sqrt{a} + \dots \text{ times } n < \frac{1 + \sqrt{(4a+1)}}{2},$$

where a is a fixed positive number and 'n' radical signs are taken on L.H.S.

34. In a triangle ABC , prove that $a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B) = ab[ac \cos B + b \cos A] + ca[ac \cos C + c \cos A] + bc[bc \cos C + c \cos B]$.

35. If $\frac{\sin^4 \alpha}{a} + \frac{\cos^4 \alpha}{b} = \frac{1}{a+b}$, then prove that $\frac{\sin^8 \alpha}{a^3} + \frac{\cos^8 \alpha}{b^3} = \frac{1}{(a+b)^3}$.



36. Prove that : $\cos x + \cos y + \cos z + \cos (x + y + z)$

$$= 4 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{y+z}{2} \right) \cos \left(\frac{z+x}{2} \right)$$

OR

$$\text{Prove that : } \sin \left(\frac{B-C}{2} \right) = \frac{b-c}{a} \cdot \cos \frac{A}{2}$$

SOLUTIONS

1. (d) : $P(1)$ is " $1^2 - 1 + 41$ is prime" i.e., 41 is prime, which is true. Similarly, $P(3)$ and $P(5)$ are also true. Now, $P(41)$ is " $41^2 - 41 + 41$ is prime" i.e., 41^2 is prime, which is certainly false.

2. (d) : We have, $2 \sin 15^\circ \cos 75^\circ$
 $= \sin(15^\circ + 75^\circ) + \sin(15^\circ - 75^\circ)$

$$= \sin 90^\circ - \sin 60^\circ = 1 - \frac{\sqrt{3}}{2} = \frac{2 - \sqrt{3}}{2}$$

3. (a) : Given, $\sin x = -\frac{1}{2}$

$$\Rightarrow \sin x = -\sin \left(\frac{\pi}{6} \right) \Rightarrow \sin x = \sin \left(\pi + \frac{\pi}{6} \right)$$

$$\Rightarrow \sin x = \sin \frac{7\pi}{6} \Rightarrow x = \frac{7\pi}{6}$$

4. (c) : $\cos(35^\circ + A) \cos(35^\circ - B) + \sin(35^\circ + A) \sin(35^\circ - B)$
 $= \cos[(35^\circ + A) - (35^\circ - B)] = \cos(A + B)$

5. (a) : Let $P(n) : 3^{2n} - 2n + 1 \dots (i)$
 On putting $n = 2$ in (i), we get $3^{2 \times 2} - (2 \times 2) + 1 = 78$, which is divisible by 2 only. So, $P(n)$ is divisible by only 2 for any n .

6. (d) : We have, $\tan 840^\circ = \tan(2 \times 360^\circ + 120^\circ)$
 $= \tan 120^\circ = \tan(90^\circ + 30^\circ) = -\cot 30^\circ = -\sqrt{3}$

7. (c) : We have, $5 \sin \theta \sin 8\theta = \frac{5}{2} [2 \sin \theta \sin 8\theta]$
 $= \frac{5}{2} [\cos(8\theta - \theta) - \cos(8\theta + \theta)] = \frac{5}{2} (\cos 7\theta - \cos 9\theta)$

8. (b) : $\angle C = 180^\circ - (\angle A + \angle B) = 30^\circ$

$$\text{Now, } \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b = \frac{c \sin B}{\sin C}$$

$$\Rightarrow b = \frac{\sqrt{2} \cdot \sin 105^\circ}{\sin 30^\circ} = \sqrt{2} \left(\frac{\sqrt{3} + 1}{2\sqrt{2}} \right) 2 = \sqrt{3} + 1$$

9. (b) : The condition $2^n(n-1)! < n^n$ is satisfied for $n > 2$.

10. (a) : By sine rule, we get $\frac{2}{(2/3)} = \frac{3}{\sin B}$

$$\Rightarrow \sin B = \left(3 \times \frac{2}{3} \times \frac{1}{2} \right) = 1 \therefore \angle B = 90^\circ$$

11. Consider $\sec 2000^\circ = \sec(5 \times 360^\circ + 200^\circ) = \sec 200^\circ$
 Since 200° lies in the third quadrant, so $\sec 200^\circ < 0$
 So, $\sec 2000^\circ$ is negative.

12. We have $\cos(-1710^\circ) = \cos(-1800^\circ + 90^\circ)$
 $= \cos(-5 \times 360^\circ + 90^\circ) = \cos\left((-5)2\pi + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

13. Given, $f(x) = 1 + 3 \cos 2x$
 We know that $-1 \leq \cos 2x \leq 1$ for all $x \in R$
 $\Rightarrow -3 \leq 3 \cos 2x \leq 3 \Rightarrow -2 \leq 1 + 3 \cos 2x \leq 4$
 $\therefore$ Range of function $f = [-2, 4]$

14. Given, $3 \cot^2 x - 1 = 0 \Rightarrow \cot^2 x = \frac{1}{3}$

$$\Rightarrow \tan^2 x = 3 = \tan^2 \frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z.$$

15. $\left(\frac{11}{16} \right)$ radians $= \left(\frac{180}{\pi} \times \frac{11}{16} \right)^\circ = \left(\frac{180}{22} \times 7 \times \frac{11}{16} \right)^\circ$
 $= \left(\frac{315}{8} \right)^\circ = \left(39 \frac{3}{8} \right)^\circ = 39^\circ \left(\frac{3}{8} \times 60 \right)''$
 $= 39^\circ 22' \left(\frac{1}{2} \times 60 \right)'' = 39^\circ 22' 30'' \text{ (nearly).}$

16. We have, $\cot \left(-\frac{15\pi}{4} \right) = -\cot \left(\frac{15\pi}{4} \right)$

$$= -\cot \left(4\pi - \frac{\pi}{4} \right) = -\left(-\cot \frac{\pi}{4} \right) = 1$$

17. L.H.S. $= \cos \left(\frac{\pi}{4} + x \right) + \cos \left(\frac{\pi}{4} - x \right)$
 $= \left(\cos \frac{\pi}{4} \cos x - \sin \frac{\pi}{4} \sin x \right) + \left(\cos \frac{\pi}{4} \cos x + \sin \frac{\pi}{4} \sin x \right)$
 $= 2 \cos \frac{\pi}{4} \cos x = 2 \times \frac{1}{\sqrt{2}} \times \cos x = \sqrt{2} \cos x = \text{R.H.S.}$

18. R.H.S. $= \frac{\sin A - \sin C}{\cos C - \cos A}$

$$= \frac{2 \sin \frac{A-C}{2} \cos \frac{A+C}{2}}{2 \sin \frac{A+C}{2} \sin \frac{A-C}{2}} = \cot \left(\frac{A+C}{2} \right)$$

$$= \cot B = \text{L.H.S.} [\because A, B, C \text{ are in A.P. } \therefore 2B = A + C]$$





19. Given $\tan^3 x - 3 \tan x = 0$

$$\Rightarrow \tan x(\tan^2 x - 3) = 0 \Rightarrow \tan x = 0 \text{ or } \tan^2 x = 3$$

$$\Rightarrow \tan x = 0 \text{ or } \tan x = \pm \sqrt{3}$$

$$\Rightarrow x = n\pi; x = n\pi \pm \frac{\pi}{3}, \text{ where } n \in \mathbb{Z}$$

20. $\tan \frac{13\pi}{12} = \tan \left(\pi + \frac{\pi}{12} \right) = \tan \frac{\pi}{12} = \tan 15^\circ$

$$= \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

21. Let $P(n)$ be the statement : $(ab)^n = a^n b^n$.

Now, $P(1) : (ab)^1 = a^1 b^1$ i.e., $ab = ab$, which is true

$\Rightarrow P(1)$ is true.

Let $P(m)$ be true i.e., $(ab)^m = a^m b^m$... (i)

For $P(m+1) : (ab)^{m+1} = (ab)^m \cdot (ab)^1 = a^m b^m ab$ [Using (i)]

$$= (a^m \cdot a)(b^m \cdot b) = a^{m+1} b^{m+1}$$

$\Rightarrow P(m+1)$ is true.

Hence, by induction, $P(n)$ is true for all $n \in \mathbb{N}$.

22. Let AB be the diameter of moon and d be the estimated diameter of moon. Then, $d = \text{arc } AB$

Now, $\theta = \left(\frac{1}{2} \right)^\circ = \left(\frac{1}{2} \times \frac{\pi}{180} \right)^\circ$ and

$$r = 360000 \text{ km}$$

$$\therefore \theta = \frac{\text{arc}}{\text{radius}}$$

$$\Rightarrow \frac{1}{2} \times \frac{\pi}{180} = \frac{d}{360000}$$

$$\Rightarrow d = \frac{1}{2} \times \frac{\pi}{180} \times 360000 = 1000\pi = 3141.6 \text{ km}$$

Hence, diameter of moon is 3141.6 km.

OR

Let the three angles be $3x$, $4x$ and $5x$.

$$\therefore 3x + 4x + 5x = 180^\circ \Rightarrow 12x = 180 \Rightarrow x = 15^\circ.$$

$\therefore$ The smallest angle = $3x = 3 \times 15^\circ = 45^\circ$ and the greatest angle = $5x = 5 \times 15^\circ = 75^\circ$

Now $75^\circ = \left(75 \times \frac{\pi}{180} \right) \text{ radians} = \frac{5\pi}{12} \text{ radians.}$

23. We have, $a \cos \theta - b \sin \theta = c$

$$\Rightarrow (a \cos \theta - b \sin \theta)^2 = c^2 \quad [\text{Squaring both sides}]$$

$$\Rightarrow a^2 \cos^2 \theta + b^2 \sin^2 \theta - 2ab \cos \theta \cdot \sin \theta = c^2$$

$$\Rightarrow a^2 (1 - \sin^2 \theta) + b^2 (1 - \cos^2 \theta) - 2ab \cos \theta \cdot \sin \theta = c^2$$

$$\Rightarrow a^2 - a^2 \sin^2 \theta + b^2 - b^2 \cos^2 \theta - 2ab \cos \theta \cdot \sin \theta = c^2$$

$$\Rightarrow a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \cos \theta \cdot \sin \theta = a^2 + b^2 - c^2$$

$$\Rightarrow (a \sin \theta + b \cos \theta)^2 = a^2 + b^2 - c^2$$

$$\Rightarrow a \sin \theta + b \cos \theta = \pm \sqrt{a^2 + b^2 - c^2}$$

24. Given, $\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$

$$\Rightarrow \cos \alpha \cos \beta - \sin \alpha \sin \beta = 1$$

$$\Rightarrow \cos(\alpha + \beta) = 1$$

Now, $1 + \cot \alpha \tan \beta = 1 + \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \beta}{\cos \beta}$

$$= \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin(\alpha + \beta)}{\sin \alpha \cos \beta}$$

$$= 0 \quad [\because \sin^2(\alpha + \beta) = 1 - \cos^2(\alpha + \beta) = 1 - 1 = 0]$$

OR

Given, $\sec x - 1 = (\sqrt{2} - 1) \tan x$

$$\Rightarrow \sec x - 1 = \tan \frac{\pi}{8} \tan x \quad \left(\because \sqrt{2} - 1 = \tan \frac{\pi}{8} \right)$$

$$\Rightarrow \frac{1}{\cos x} - 1 = \frac{\sin \frac{\pi}{8}}{\cos \frac{\pi}{8}} \cdot \frac{\sin x}{\cos x} \cdot \cos x \neq 0$$

$$\Rightarrow \cos \frac{\pi}{8} - \cos x \cos \frac{\pi}{8} = \sin x \sin \frac{\pi}{8}$$

CBSE Class X: 2.3 lakh get 90%, 95 percenters double

The dizzying upward march of marks in board exams continued when the CBSE Class X exam results were declared. Of the 17.6 lakh students who appeared for the tests, a record 2.25 lakh scored 90% and above, with as many as 57,256 scoring 95% and above in aggregate.

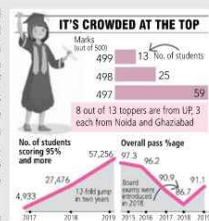
The number of high scorers nearly doubled from 2018, the year when the Class X Board made a comeback.

As against four all-India toppers last year, there were 13 students who topped scoring 499 out of 500, followed by 25 students who got 498 and 59 who scored 497. The results were announced 32 days after the completion of the exam.

The overall pass percentage improved by 4.4% point, recording 91.1%, which is also the highest in the history of compulsory Class X Board exams for CBSE. The last compulsory Class X exam before the CCE era was in 2009 when the pass percentage was 88.84%. In 2018, Class X Boards made a comeback with a pass percentage of 86.7%.

Girls continued to outshine boys with 92.45% pass percentage as against 90.14% of the boys. However, the gap between girls and boys narrowed from 3.4 percentage points last year to 2.3 percentage points. A total of 17 lakh students appeared for the exam this year, an increase of one lakh over 2018.

Courtesy: The Times of India





$$\Rightarrow \cos \frac{\pi}{8} = \cos x \cos \frac{\pi}{8} + \sin x \sin \frac{\pi}{8}$$

$$\Rightarrow \cos \left(x - \frac{\pi}{8} \right) = \cos \frac{\pi}{8} \Rightarrow x - \frac{\pi}{8} = 2n\pi \pm \frac{\pi}{8}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{4} \text{ or } x = 2n\pi, n \in \mathbb{Z}$$

25. Given, $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$
 $\therefore (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$
 $\Rightarrow (\cos^2 \alpha - \sin^2 \alpha) + (\cos^2 \beta - \sin^2 \beta) + 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta = 0$
 $\Rightarrow \cos 2\alpha + \cos 2\beta + 2 \cos(\alpha + \beta) = 0$

26. We have, $\sin \theta = n \sin(\theta + 2\alpha)$
 $\Rightarrow \frac{\sin(\theta + 2\alpha)}{\sin \theta} = \frac{1}{n} \Rightarrow \frac{\sin(\theta + 2\alpha) + \sin \theta}{\sin(\theta + 2\alpha) - \sin \theta} = \frac{1+n}{1-n}$
 $\Rightarrow \frac{2 \sin(\theta + \alpha) \cos \alpha}{2 \sin \alpha \cos(\theta + \alpha)} = \frac{1+n}{1-n}$
 $\Rightarrow \tan(\theta + \alpha) = \left(\frac{1+n}{1-n} \right) \tan \alpha$

27. Given, $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$
 $\Rightarrow \frac{\sin(\pi \cos \theta)}{\cos(\pi \cos \theta)} = \frac{\cos(\pi \sin \theta)}{\sin(\pi \sin \theta)}$
 $\Rightarrow \sin(\pi \cos \theta) \sin(\pi \sin \theta) = \cos(\pi \sin \theta) \cos(\pi \cos \theta)$
 $\Rightarrow \cos(\pi \cos \theta) \cos(\pi \sin \theta) - \sin(\pi \cos \theta) \sin(\pi \sin \theta) = 0$
 $\Rightarrow \cos(\pi \cos \theta + \pi \sin \theta) = 0$

$$\Rightarrow \pi \cos \theta + \pi \sin \theta = \pm \frac{\pi}{2} \quad \left[\because \cos \left(\pm \frac{\pi}{2} \right) = 0 \right]$$

$$\Rightarrow \cos \theta + \sin \theta = \pm \frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta = \pm \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4} = \pm \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos \left(\theta - \frac{\pi}{4} \right) = \pm \frac{1}{2\sqrt{2}}$$

28. Let $P(n) : 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + n \times n! = (n+1)! - 1$
 Now, $P(1) : 1 \times 1! = (1+1)! - 1 \Rightarrow 1 = 1$, which is true.
 Let $P(m)$ be true
 i.e., $1 \times 1! + 2 \times 2! + 3 \times 3! + \dots + m \times m! = (m+1)! - 1$... (i)

Now, we have to show that $P(m+1)$ is true i.e.,
 $1 \times 1! + 2 \times 2! + \dots + m \times m! + (m+1) \times (m+1)! = (m+2)! - 1$
 L.H.S. $= 1 \times 1! + 2 \times 2! + \dots + m \times m! + (m+1) \times (m+1)!$
 $= (m+1)! - 1 + (m+1) \times (m+1)! \quad [\text{Using (i)}]$
 $= (m+1)! (m+2) - 1 = (m+2)! - 1 = \text{R.H.S.}$

$\Rightarrow P(n+1)$ is true.

Hence by mathematical induction, $P(n)$ is true for all $n \in \mathbb{N}$.

OR

Refer to answer 14, Page no. 86 of MTG CBSE Champion Mathematics, Class-11

29. Given, $\tan \theta = \frac{m}{n}, 0 < \theta < \frac{\pi}{2}$
 Now, $\sec \theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + \left(\frac{m}{n} \right)^2} = \frac{\sqrt{m^2 + n^2}}{n}$

$$\Rightarrow \cos \theta = \frac{n}{\sqrt{m^2 + n^2}}$$

Also, $\sin \theta = \tan \theta \cdot \cos \theta = \frac{m}{n} \cdot \frac{n}{\sqrt{m^2 + n^2}} = \frac{m}{\sqrt{m^2 + n^2}}$

Now, L.H.S. $= \frac{m}{\sin \phi} - \frac{n}{\cos \phi} = \frac{m}{\frac{m}{\sqrt{m^2 + n^2}}} - \frac{n}{\frac{n}{\sqrt{m^2 + n^2}}} = \frac{m}{\sin \phi} - \frac{n}{\cos \phi}$
 $= \frac{\sqrt{m^2 + n^2}}{\sin \phi \cos \phi} \left(\sin \theta \cos \phi - \cos \theta \sin \phi \right)$
 $= 2\sqrt{m^2 + n^2} \cdot \frac{\sin(\theta - \phi)}{2 \sin \phi \cos \phi} = 2\sqrt{m^2 + n^2} \cdot \frac{\sin(3\phi - \phi)}{\sin 2\phi}$
 $= 2\sqrt{m^2 + n^2} \cdot \frac{\sin 2\phi}{\sin 2\phi} = 2\sqrt{m^2 + n^2} = \text{R.H.S.}$

30. We have, $a \tan \theta + b \sec \theta = c$... (i)
 $\Rightarrow c - a \tan \theta = b \sec \theta$
 $\Rightarrow (c - a \tan \theta)^2 = b^2 \sec^2 \theta$
 $\Rightarrow c^2 + a^2 \tan^2 \theta - 2ac \tan \theta = b^2 (1 + \tan^2 \theta)$
 $\Rightarrow \tan^2 \theta (a^2 - b^2) - 2ac \tan \theta + (c^2 - b^2) = 0$... (ii)
 It is given that α and β satisfy equation (i). Therefore, $\tan \alpha$ and $\tan \beta$ are roots of (ii).

$$\therefore \tan \alpha + \tan \beta = \frac{2ac}{a^2 - b^2} \text{ and } \tan \alpha \tan \beta = \frac{c^2 - b^2}{a^2 - b^2}$$

Now, $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{2ac}{a^2 - b^2}}{1 - \frac{c^2 - b^2}{a^2 - b^2}} = \frac{2ac}{a^2 - c^2}$

OR

Given, $f(x) = \frac{\cot x}{1 + \cot x} = \frac{1}{\frac{1}{\cot x} + 1} = \frac{1}{\tan x + 1}$

Now, $f(\alpha) f(\beta) = \frac{1}{1 + \tan \alpha} \cdot \frac{1}{1 + \tan \beta}$
 $= \frac{1}{1 + \tan \alpha} \cdot \frac{1}{1 + \tan \left(\frac{5\pi}{4} - \alpha \right)} \quad \left[\because \alpha + \beta = \frac{5\pi}{4} \text{ (Given)} \right]$



$$\begin{aligned}
 &= \frac{1}{1+\tan \alpha} \cdot \frac{1}{1+\tan\left(\frac{\pi}{4}-\alpha\right)} \\
 &\quad \left[\because \tan\left(\frac{5\pi}{4}-\alpha\right) = \tan\left(\pi + \frac{\pi}{4}-\alpha\right) = \tan\left(\frac{\pi}{4}-\alpha\right) \right] \\
 &= \frac{1}{1+\tan \alpha} \cdot \frac{1}{1+\frac{1-\tan \alpha}{1+\tan \alpha}} \\
 &= \frac{1}{1+\tan \alpha} \cdot \frac{1+\tan \alpha}{(1+\tan \alpha)+(1-\tan \alpha)} = \frac{1}{2}
 \end{aligned}$$

31. L.H.S. = $\cos 5A = \cos(3A + 2A)$
 $= \cos 3A \cos 2A - \sin 3A \sin 2A$
 $= (4 \cos^3 A - 3 \cos A)(2 \cos^2 A - 1) - (3 \sin A - 4 \sin^3 A)(2 \sin A \cos A)$
 $= (4 \cos^3 A - 3 \cos A)(2 \cos^2 A - 1) - (3 \sin A - 4 \sin^3 A)(2 \sin A \cos A)$
 $= (4 \cos^3 A - 3 \cos A)(2 \cos^2 A - 1) - [3 - 4(1 - \cos^2 A)] [2(1 - \cos^2 A) \cos A]$
 $= (8 \cos^5 A - 10 \cos^3 A + 3 \cos A) - 2 \cos A(1 - \cos^2 A)(4 \cos^2 A - 1)$
 $= (8 \cos^5 A - 10 \cos^3 A + 3 \cos A) - 2 \cos A(5 \cos^2 A - 4 \cos^4 A - 1)$
 $= 16 \cos^5 A - 20 \cos^3 A + 5 \cos A = \text{R.H.S.}$

32. Consider $\sin x + \cos x = \sqrt{2} \left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right)$
 $= \sqrt{2} \left(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} \right) = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) \leq \sqrt{2}$
 $\left(\because \sin \left(x + \frac{\pi}{4} \right) \leq 1 \text{ for all } x \in \mathbb{R} \right)$

But $\sqrt{2} < \frac{\pi}{2}$ ($\because \sqrt{2} = 1.414$ nearly and $\frac{\pi}{2} = 1.571$ nearly)

$\therefore \sin x + \cos x < \frac{\pi}{2} \Rightarrow \cos x < \frac{\pi}{2} - \sin x$
 $\Rightarrow \sin(\cos x) < \sin\left(\frac{\pi}{2} - \sin x\right)$
 $\left(\because \ln\left(0, \frac{\pi}{2}\right) \text{ if } \alpha < \beta, \text{ then } \sin \alpha < \sin \beta \right)$

$\Rightarrow \sin(\cos x) < \cos(\sin x) \Rightarrow \cos(\sin x) > \sin(\cos x)$

33. Let $P(n) : 7 + 77 + 777 + \dots$ to n terms
 $= \frac{7}{81}(10^{n+1} - 9n - 10)$

For $n=1, 7 = \frac{7}{81}(10^2 - 9 \times 1 - 10)$ i.e., $7 = \frac{7}{81} \times 81$ i.e., $7 = 7$,
 which is true.

$\therefore P(1)$ is true.

Let $P(m)$ be true i.e., $7 + 77 + 777 + \dots$ to m terms

$= \frac{7}{81}(10^{m+1} - 9m - 10)$... (i)

Now, we have to show that $P(m+1)$ is true i.e.,
 $7 + 77 + 777 + \dots$ to $(m+1)$ terms

$= \frac{7}{81}[10^{m+2} - 9(m+1) - 10]$

Consider $(m+1)^{\text{th}}$ term = $777 \dots 7$ ($m+1$) digits
 $= 7 + 7 \cdot 10^1 + 7 \cdot 10^2 + \dots$ to $(m+1)$ terms

$= 7 \left(\frac{10^{m+1} - 1}{10 - 1} \right) = \frac{7}{9}(10^{m+1} - 1)$... (ii)

Now, $7 + 77 + 777 + \dots$ to $(m+1)$ terms
 $= (7 + 77 + 777 + \dots$ to m terms) $+ (m+1)^{\text{th}}$ term

$= \frac{7}{81}(10^{m+1} - 9m - 10) + \frac{7}{9}(10^{m+1} - 1)$ [Using (i) and (ii)]

$= \frac{7}{81}(10^{m+1} - 9m - 10 + 9 \cdot 10^{m+1} - 9)$

$= \frac{7}{81}[(1+9) \times 10^{m+1} - 9(m+1) - 10]$

$= \frac{7}{81}[10^{m+2} - 9(m+1) - 10]$

$\therefore P(m+1)$ is true.

Hence, by induction, the given statement is true for $n \in \mathbb{N}$.

OR

Refer to answer 37, Page no. 92 of MTG CBSE Champion Mathematics, Class-11

34. Consider $a^3 \cos(B - C) = a^2 [a \cos(B - C)]$
 $= a^2 [k \sin A \cdot \cos(B - C)] = a^2 \cdot k [\sin A \cos(B - C)]$
 $= a^2 \cdot k [\sin(B + C) \cos(B - C)]$ [$\because A = 180^\circ - (B + C)$]
 $= a^2 \cdot \frac{k}{2} [2 \sin(B + C) \cos(B - C)]$
 $= a^2 \cdot \frac{k}{2} [\sin 2B + \sin 2C]$

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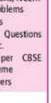
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HIGHLIGHTS

- Important Facts/Formulae & Comprehensive Theory
- Practice Questions, NCERT & Exemplar Problems
- HOTS Questions
- Previous Years' Questions of KVS, NCT, etc.
- Answers as per CBSE Marking Scheme
- 10 Practice Papers



$$= a^2 \cdot \frac{k}{2} [2 \sin B \cos B + 2 \sin C \cos C]$$

$$= a^2 \cdot [b \cos B + c \cos C] \quad \dots(i)$$

$$[\because k \sin B = b \text{ and } k \sin C = c]$$

$$\text{Similarly, } b^2 \cos(C - A) = b^2 [c \cos C + a \cos A] \quad \dots(ii)$$

$$\text{and } c^2 \cos(A - B) = c^2 [a \cos A + b \cos B] \quad \dots(iii)$$

Adding (i), (ii) and (iii), we get

$$[a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B)]$$

$$= a^2 b \cos B + a^2 c \cos C + b^2 c \cos C + b^2 a \cos A$$

$$+ c^2 a \cos A + c^2 b \cos B$$

$$= (a^2 b \cos B + b^2 a \cos A) + (ca^2 \cos C + ac^2 \cos A) +$$

$$(b^2 c \cos C + c^2 b \cos B)$$

$$= ab [a \cos B + b \cos A] + ca [a \cos C + c \cos A] +$$

$$bc [b \cos C + c \cos B]$$

35. Given, $\frac{\sin^4 \alpha}{a} + \frac{\cos^4 \alpha}{b} = \frac{1}{a+b}$

$$\Rightarrow \frac{\sin^4 \alpha}{a} + \frac{(1 - \sin^2 \alpha)^2}{b} = \frac{1}{a+b}$$

$$\Rightarrow b \sin^4 \alpha + a(1 + \sin^4 \alpha - 2 \sin^2 \alpha) = \frac{ab}{a+b}$$

$$\Rightarrow (a+b) \sin^4 \alpha - 2a \sin^2 \alpha + a = \frac{ab}{a+b}$$

$$\Rightarrow (a+b)^2 \sin^4 \alpha - 2a(a+b) \sin^2 \alpha + a^2 + ab = ab$$

$$\Rightarrow (a+b)^2 \sin^4 \alpha - 2a(a+b) \sin^2 \alpha + a^2 = 0$$

$$\Rightarrow [(a+b) \sin^2 \alpha - a]^2 = 0 \Rightarrow (a+b) \sin^2 \alpha - a = 0$$

$$\Rightarrow \sin^2 \alpha = \frac{a}{a+b} \quad \dots(i)$$

$$\text{Also, } \cos^2 \alpha = 1 - \sin^2 \alpha = 1 - \frac{a}{a+b} = \frac{b}{a+b} \quad \dots(ii)$$

$$\text{Now, } \frac{\sin^8 \alpha}{a^3} + \frac{\cos^8 \alpha}{b^3} = \frac{1}{a^3} \left(\frac{a}{a+b} \right)^4 + \frac{1}{b^3} \left(\frac{b}{a+b} \right)^4$$

$$[\text{Using (i) and (ii)}]$$

$$= \frac{a^4}{a^3(a+b)^4} + \frac{b^4}{b^3(a+b)^4} = \frac{a}{(a+b)^4} + \frac{b}{(a+b)^4}$$

$$= \frac{a+b}{(a+b)^4} = \frac{1}{(a+b)^3}$$

36. L.H.S. = $\cos x + \cos y + \cos z + \cos(x+y+z)$

$$= (\cos x + \cos y) + [\cos z + \cos(x+y+z)]$$

$$= 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$+ 2 \cos \left(\frac{z+x+y+z}{2} \right) \cos \left(\frac{z-x-y-z}{2} \right)$$

$$= 2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$$

$$+ 2 \cos \left(\frac{x+y+2z}{2} \right) \cos \left(\frac{x+y}{2} \right)$$

$$= 2 \cos \left(\frac{x+y}{2} \right) \left[\cos \left(\frac{x-y}{2} \right) + \cos \left(\frac{x+y+2z}{2} \right) \right]$$

$$= 2 \cos \left(\frac{x+y}{2} \right) \left[2 \cos \left(\frac{\frac{x-y}{2} + \frac{x+y+2z}{2}}{2} \right) \times \right.$$

$$\left. \cos \left(\frac{\frac{x+y+2z}{2} - \frac{x-y}{2}}{2} \right) \right]$$

$$= 2 \cos \left(\frac{x+y}{2} \right) \left[2 \cos \left(\frac{x+z}{2} \right) \cos \left(\frac{y+z}{2} \right) \right]$$

$$= 4 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{y+z}{2} \right) \cos \left(\frac{x+z}{2} \right) = \text{R.H.S.}$$

OR

Refer to answer 146, Page no. 78 of MTG CBSE Champion Mathematics, Class-11



CBSE Class X exam to reward 'creativity'

The Central Board of Secondary Education is planning some big changes in the pattern of questions for the Class X exams, from the current academic year, to reward "creative answers and discourage rote learning".

As part of its ongoing reforms, CBSE has decided to diversify the current format of objective questions that carry 20 marks. But the bigger changes will happen in the theory portion worth 60 marks. These sections will have fewer questions, each with higher marks, to encourage long form and creative answers.

The internal assessment done by schools, which accounts for the remaining 20 marks, will henceforth require students to create a portfolio that will carry 5-10 marks.

A CBSE official said to ensure quality in the objective questions carrying one mark each, different methods of obtaining the answer will be employed. While some will have multiple choice answers, others will need blanks to be filled in or the answer given in a single sentence.

CBSE will reduce the number of questions in the portion carrying 60 marks. Each question will now carry more marks, an official said.

"Students will have to answer in the long form, longer than in the present format, and in a more creative way. The marking scheme and evaluation will be formulated likewise," the official said. For the practical/internal assessment segment of 20 marks, students will have to prepare a portfolio on a particular subject and their learning during the year.

"It has not yet been decided how many marks this will carry, but a minimum of five marks will be allocated for the portfolio and could be up to 10 marks as well," added the official.

According to CBSE sources, the sample question papers based on the new pattern will be made available by the board on its website from July 2019.

"We will share the new pattern in the form of sample papers for teachers and students for their understanding and preparation," disclosed a CBSE official.



MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 1: Sets, Relations and Functions

Time Taken : 60 Min.

Only One Option Correct Type

- The domain of the function $f(x) = \sqrt{2-|x|} + \sqrt{1+|x|}$ is
(a) [2, 6] (b) [-2, 6]
(c) [8, 12] (d) [-2, 2]
- If A and B are two sets, then $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to
(a) $A \cup B$ (b) $A \cap B$
(c) A (d) B'
- In a class, 70 students wrote two tests viz; test-I and test-II. 50% of the students failed in test-I and 40% of the students in test-II. How many students passed in both tests?
(a) 21 (b) 7 (c) 28 (d) 14
- If $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1 \forall x \in \mathbb{R} - \{0\}$, then $5x^2f(x^2)$ is
(a) $(1 + 2x^2)(1 - 3x^2)$ (b) $(3 - 2x^2)(1 + x^2)$
(c) $(3 + 2x^2)(1 - x^2)$ (d) none of these
- On its annual sports day, school awarded 35 medals in athletics, 15 in judo and 18 in swimming. If these medals goes to a total of 58 students and only three of them got medals in all the three sports. The number of students who received medals in exactly two of the three sports are
(a) 9 (b) 4 (c) 5 (d) 7
- If $A = \{7, 8, 9\}$ and $B = \{9, 5\}$, then $(A \cup B) \times (A \cap B)$ is
(a) $\{(7, 9), (7, 5), (8, 9), (8, 5), (9, 9), (9, 5)\}$
(b) $\{(5, 9), (7, 9), (8, 9), (9, 9)\}$

- (c) $\{(9, 5), (9, 7), (9, 8), (9, 9)\}$
(d) none of these

One or More Than One Option(s) Correct Type

- From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is
(a) 11 (b) 12 (c) 13 (d) 14

SOLUTION SENDERS

Samurai Sudoku (May)

- Shailaja Tekaday (Maharashtra)

Mathdoku (May)

- Devjit Acharjee (West Bengal)
- Arkamitra Roy (West Bengal)
- Sulagna Nandimondal (West Bengal)
- Rajashree Tekaday (Maharashtra)

Maths Musing Set - 197

- Gomati Goswami (Mumbai)
- Devjit Acharjee (West Bengal)
- Gouri Sankar Adhikary (West Bengal)
- Khokon Kumar Nandi (West Bengal)
- Jayanti Narayanan



8. If $e^x + e^{f(x)} = e$, then for $f(x)$
 (a) domain = $(-\infty, 1)$ (b) range = $(-\infty, 1)$
 (c) domain = $(-\infty, 0]$ (d) range = $(-\infty, 1]$
9. If $f(x) = \frac{x}{x^2 + 1}$ and $f(A) = \left\{ y : -\frac{1}{2} \leq y < 0 \right\}$, then set A is
 (a) $[-1, 0]$ (b) $(-\infty, -1]$
 (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$
10. If $A = \{(x, y) : x^2 + y^2 \leq 1, x, y \in R\}$ and $B = \{(x, y) : x^2 + y^2 \leq 4, x, y \in R\}$, then
 (a) $A - B = A$ (b) $B - A = B$
 (c) $A - B = \emptyset$ (d) $B - A \neq \emptyset$
11. Let N be the set of natural numbers and for $a \in N$, aN denotes the set $\{ax : x \in N\}$. If $bN \cap cN = dN$, where b, c, d are natural numbers greater than 1 and the greatest common divisor (G.C.D.) of b and c is 1, then d equals
 (a) $b + c$ (b) $\{b, c\}$
 (c) $\min\{b, c\}$ (d) bc
12. Let Z denote the set of integers, then
 $\{x \in Z : |x - 3| < 4\} \cap \{x \in Z : |x - 4| < 5\} =$
 (a) $\{-1, 0, 1, 2, 3, 4\}$ (b) $\{-1, 0, 1, 2, 3, 4, 5\}$
 (c) $\{0, 1, 2, 3, 4, 5, 6\}$
 (d) $\{-1, 0, 1, 2, 3, 5, 6, 7, 8, 9\}$
13. Let $A = \{a, b\}$, $B = \{c, d\}$ and $f : A \rightarrow B$ given by $f(x) = 3x^2 - 2$ and $g : A \rightarrow B$ given by $g(x) = 1 - 8x$, if $f = g$, then
 (a) $A = \left\{-2, \frac{1}{2}\right\}$ (b) $B = \left\{24, \frac{-23}{3}\right\}$
 (c) $A = \left\{-3, \frac{1}{3}\right\}$ (d) $B = \left\{25, \frac{-5}{3}\right\}$

Comprehension Type

In a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18.

14. How many students have opted Mathematics alone?
 (a) 60 (b) 35 (c) 70 (d) 48
15. How many students have opted Chemistry alone?
 (a) 7 (b) 15 (c) 12 (d) 18

Matrix Match Type

16. Match the following :

	Column-I	Column-II
P.	The range of the function $f(x) = \frac{1}{2 - \cos 3x}$ is	1. a subset of $[0, 1]$
Q.	The range of the function $f(x) = \sqrt{(x-4)} + \sqrt{(6-x)}$ is	2. a subset of $[1, 2]$
R.	If the function $f(x)$ is defined for $x \in [0, 1]$, then the function $f(2x+3)$ is defined for	3. $\left[-\frac{3}{2}, -1\right]$
		4. $\left[\frac{1}{3}, 1\right]$
		5. $[\sqrt{2}, 2]$

P	Q	R
(a) 1	2, 4	3, 5
(b) 1, 4	2, 3	5
(c) 1, 4	2, 5	3
(d) 4	2, 5	1, 3

Numerical Answer Type

17. A polynomial $f(x)$ satisfies the condition $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(10) = 1001$, then the value of $f(20)$ must be
18. If $n(A \cap B) = 10$, $n(B \cap C) = 20$ and $n(A \cap C) = 30$, then the greatest possible value of $n(A \cap B \cap C)$ is
19. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
20. If $f(x) = a(x^n + 3)$ and $f(1) = 12$, $f(3) = 36$, then the value of $8f(2)$ must be



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SELF CHECK

No. of questions attempted
 No. of questions correct
 Marks scored in percentage

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY !	Revise thoroughly and strengthen your concepts.



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1. For what values of k will the equation $|z - z_1|^2 + |z - z_2|^2 = k, k \in \mathbb{R}$, represent a circle?

(Ayush Garg, U.P.)

Ans. The given equation can be written as

$$\begin{aligned} 2|z|^2 + |z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z\bar{z}_1) - 2\operatorname{Re}(z\bar{z}_2) &= k \\ \Rightarrow 2|z|^2 - 2\operatorname{Re}(z(\bar{z}_1 + \bar{z}_2)) &= k - (|z_1|^2 + |z_2|^2) \\ \Rightarrow |z|^2 - \operatorname{Re}(z(\bar{z}_1 + \bar{z}_2)) &= \frac{1}{2}(k - |z_1|^2 - |z_2|^2) \\ \Rightarrow |z|^2 - 2\operatorname{Re}\left\{z\left(\frac{\bar{z}_1 + \bar{z}_2}{2}\right)\right\} &= \frac{1}{2}(k - |z_1|^2 - |z_2|^2) \\ \Rightarrow \left|z - \frac{z_1 + z_2}{2}\right|^2 - \frac{1}{4}|z_1 + z_2|^2 &= \frac{1}{2}(k - |z_1|^2 - |z_2|^2) \\ \Rightarrow \left|z - \frac{z_1 + z_2}{2}\right|^2 &= \frac{1}{2}k - \frac{1}{4}(|z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z_1\bar{z}_2)) \\ &= \frac{1}{2}k - \frac{1}{4}|z_1 - z_2|^2 \\ \Rightarrow \left|z - \frac{z_1 + z_2}{2}\right|^2 &= \frac{1}{4}(2k - |z_1 - z_2|^2), \end{aligned}$$

which will represent a real circle having centre at $\frac{z_1 + z_2}{2}$ and radius $\frac{1}{2}\sqrt{2k - |z_1 - z_2|^2}$ provided $k \geq \frac{1}{2}|z_1 - z_2|^2$.

2. The sum of the squares of three distinct real numbers in G.P., is S^2 . If their sum is λS , show that

$$\lambda^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3). \quad (\text{Shashank Roy, West Bengal})$$

Ans. Let three numbers in G.P. be a, ar, ar^2 .

According to the given condition, we have

$$\begin{aligned} \lambda S &= a + ar + ar^2 & \dots (i) \\ \text{and } S^2 &= a^2 + a^2r^2 + a^2r^4 & \dots (ii) \end{aligned}$$

Squaring equation (i) and dividing by equation (ii), we have

$$\begin{aligned} \lambda^2 &= \frac{a^2(1+r+r^2)^2}{a^2(1+r^2+r^4)} = \left(\frac{1-r^3}{1-r}\right) \times \left(\frac{1-r^2}{1-r^6}\right) \\ &= \frac{1-r^3}{1-r} \times \frac{1+r}{1+r^3} = \frac{1+r+r^2}{1-r+r^2} \\ \therefore (\lambda^2 - 1)r^2 - (\lambda^2 + 1)r + (\lambda^2 - 1) &= 0 \quad \dots (iii) \end{aligned}$$

For real r , we have

$$(\lambda^2 + 1)^2 > 4(\lambda^2 - 1)^2 \text{ and } (\lambda^2 - 1) \neq 0$$

Solving the first inequality, we have $x^2 > 4(x-2)^2$

$$[\text{Putting } \lambda^2 + 1 = x]$$

$$\Rightarrow 3x^2 - 16x + 16 < 0$$

$$\Rightarrow (x-4)\left(x-\frac{4}{3}\right) < 0 \Rightarrow \frac{4}{3} < x < 4$$

$$\Rightarrow \frac{4}{3} < \lambda^2 + 1 < 4 \Rightarrow \frac{1}{3} < \lambda^2 < 3 \Rightarrow \lambda^2 \in \left(\frac{1}{3}, 3\right)$$

From second inequality $\lambda^2 \neq 1$

$$\therefore \lambda^2 \in \left(\frac{1}{3}, 1\right) \cup (1, 3)$$

3. Let $[\sqrt{n^2+1}] = [\sqrt{n^2+\lambda}]$, where $n, \lambda \in \mathbb{N}$ and $[\cdot]$ represents greatest integer function. Show that λ can have $2n$ different values. (Tejaswin Jha, Bihar)

Ans. We have, $n^2 + 1 = (n+1)^2 - 2n < (n+1)^2 \forall n \in \mathbb{N}$

$$\Rightarrow \sqrt{n^2+1} < n+1$$

Thus, we have, $n < \sqrt{n^2+1} < n+1$

$$\therefore [\sqrt{n^2+1}] = n$$

Now, the given equation reduces to $[\sqrt{n^2+\lambda}] = n$

$$\Rightarrow n < \sqrt{n^2+\lambda} < (n+1) \Rightarrow n^2 < n^2+\lambda < (n+1)^2 \Rightarrow 0 < \lambda < 2n+1.$$

Hence, λ can have $2n$ different values.

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MATHS MUSING

SOLUTION SET-197

$$\begin{aligned}
 1. \text{ (d): We have } & \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} \\
 &= \lim_{x \rightarrow 0} \frac{\log \cos x}{\log(1+x^2)} = \lim_{x \rightarrow 0} \frac{\log(1 - 1 + \cos x)}{\log(1+x^2)} \cdot \frac{1 - \cos x}{1 - \cos x} \\
 &= \lim_{x \rightarrow 0} \frac{\log[1 - (1 - \cos x)]}{1 - \cos x} \cdot \frac{1 - \cos x}{\log(1+x^2)} \\
 &= -\lim_{x \rightarrow 0} \frac{\log[1 - (1 - \cos x)]}{-(1 - \cos x)} \cdot \frac{2 \sin^2 \frac{x}{2}}{4 \left(\frac{x}{2}\right)^2 \log(1+x^2)} = -\frac{1}{2}
 \end{aligned}$$

Hence, $f(x)$ is differentiable at $x = 0$ and hence continuous at $x = 0$.

$$\begin{aligned}
 2. \text{ (b): } f(x) &= \begin{cases} e^{\cos x} \sin x, & \text{for } |x| \leq 2 \\ 2, & \text{otherwise} \end{cases} \\
 \Rightarrow \int_{-2}^3 f(x) dx &= \int_{-2}^2 f(x) dx + \int_2^3 f(x) dx \\
 &= \int_{-2}^2 e^{\cos x} \sin x dx + \int_2^3 2 dx = 0 + 2 [3 - 2] = 2
 \end{aligned}$$

$$\begin{aligned}
 3. \text{ (b): Since, } & \sum_{r=1}^n 2 \cos(r^2 x) \sin(rx) = 1 \\
 \Rightarrow \sum_{r=1}^n [\sin\{r(r+1)x\} - \sin\{r(r-1)x\}] &= 1 \\
 \Rightarrow \sin(n(n+1)x) &= 1 \\
 \therefore x &= \frac{(4m+1)\pi}{n(n+1) \cdot 2}, m \in \mathbb{I}
 \end{aligned}$$

$$\begin{aligned}
 4. \text{ (c): Since, } z &= 0 \\
 \therefore \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1} &\Rightarrow \frac{x-2}{3} = 1 \text{ and } \frac{y+1}{2} = 1 \\
 \Rightarrow x = 5 \text{ and } y = 1 \\
 \text{Putting these values in } xy = c^2, &\text{ we get} \\
 c^2 = 5 &\Rightarrow c = \pm\sqrt{5}
 \end{aligned}$$

5. (a)

$$\begin{aligned}
 6. \text{ (b, c): Let } \vec{a} &= \hat{i} + \hat{j} \\
 \text{Since, } \vec{b} \text{ and } \vec{c} &\text{ are perpendicular to each other} \\
 \therefore \vec{b} \cdot \vec{c} &= 0 \text{ or } (4\hat{i} + 3\hat{j}) \cdot (a\hat{i} + b\hat{j}) = 0 \\
 \Rightarrow 4a + 3b &= 0 \Rightarrow \frac{a}{3} = \frac{b}{-4} = k \text{ (say)}
 \end{aligned}$$

$$\therefore \vec{c} = k(3\hat{i} - 4\hat{j})$$

$$\begin{aligned}
 \text{Let } \alpha &= x\hat{i} + y\hat{j} \text{ be the required vector, then} \\
 \alpha \cdot \vec{b} &= (\text{length of } \vec{b}) (\text{projection of } \alpha \text{ on } \vec{b}) \\
 \Rightarrow 4x + 3y &= \sqrt{25} \times 1 \text{ or } 4x + 3y = 5 \quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 \text{and } \vec{c} \cdot \alpha &= (\text{length of } \vec{c}) (\text{projection of } \alpha \text{ on } \vec{c}) \\
 \Rightarrow 3kx - 4ky &= 5 |k| \times 2 \Rightarrow 3x - 4y = \pm 10 \quad \dots(ii)
 \end{aligned}$$

From (i) and (ii), $x = 2, y = -1$ and $x = -(2/5), y = 11/5$

$$\therefore \text{ The required vectors are } 2\hat{i} - \hat{j} \text{ and } \frac{(-2\hat{i} + 11\hat{j})}{5}$$

7. (b): The vector perpendicular to L_1 and L_2 is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -\hat{i} - 7\hat{j} + 5\hat{k}$$

$$\therefore \text{ Required unit vector is } \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

8. (d): The points on L_1 and L_2 are $A(-1, -2, -1)$ and $C(2, -2, 3)$

The shortest distance is

$$\overline{AC} \times \hat{n} = (3\hat{i} + 4\hat{k}) \cdot \frac{(-\hat{i} - 7\hat{j} + 5\hat{k})}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

$$9. (32): \because F(x) = \frac{1}{x^2} \int_4^x [4t^2 - 2F'(t)] dt \quad \dots(i)$$

$$\Rightarrow x^2 F(x) = \int_4^x [4t^2 - 2F'(t)] dt$$

Differentiating both sides w.r.t. x , we get

$$\begin{aligned}
 x^2 F'(x) + F(x) \cdot 2x &= 4x^2 - 2F'(x) \quad \dots(ii) \\
 \Rightarrow 16F'(4) + 8F(4) &= 64 - 2F'(4) \\
 [\text{Put } x = 4 \text{ in eqn. (ii)}] \\
 \therefore F'(4) &= 32/9 \quad [\because F(4) = 0, \text{ from eqn. (i)}]
 \end{aligned}$$

10. (b): $P \rightarrow 2, Q \rightarrow 1, R \rightarrow 2, 3$

$$\begin{aligned}
 (P) f'(x) &= 4x - \frac{1}{x} = \frac{4x^2 - 1}{x} \text{ Thus } f'(x) > 0 \text{ if } x \in (-1/2, 0) \\
 &\cup (1/2, \infty). \text{ So } f(x) \text{ increases on } (-1/2, 0) \cup (1/2, \infty). \text{ In} \\
 &\text{particular on } (2, \infty).
 \end{aligned}$$

$$(Q) f'(x) = 1 - 2 \cos x$$

Thus $f'(x)$ is periodic with period 2π and $f(x)$ decreases on $(0, \pi/3) \cup (5\pi/3, 2\pi)$ and increases on $(\pi/3, 5\pi/3)$. In particular, $f(x)$ decreases on $(0, 1/2)$.

(R) $f(x)$ decreases on $(-\infty, 0) \cup (2, \infty)$ and increases on $(0, 2)$. In particular, $f(x)$ decreases on $(-\infty, 0), (2, \infty)$.

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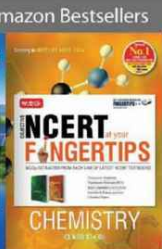
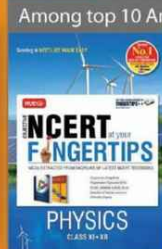
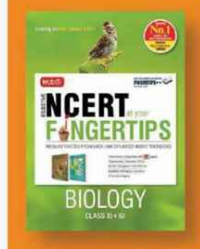
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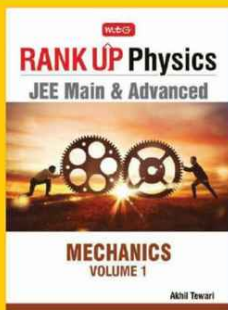
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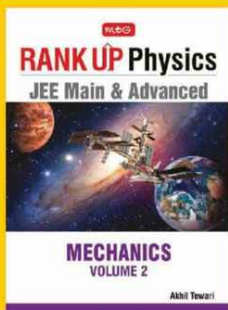
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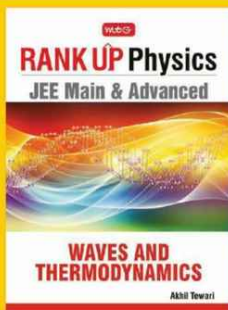
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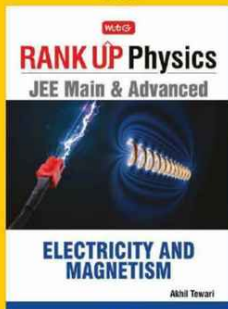
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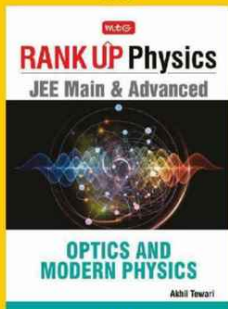
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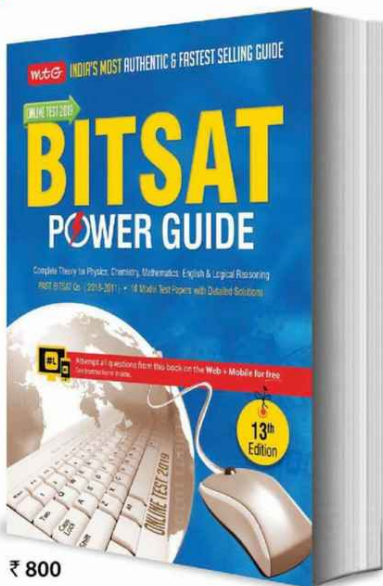
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